UNIVERSITÄT HAMBURG

Degradable networks with general up and down time distributions

Cornelia Sauer, Hans Daduna

Preprint–No. 2004-04   June 2004

FACHBEREICH MATHEMATIK

SCHWERPUNKT MATHEMATISCHE STATISTIK
UND STOCHASTISCHE PROZESSE

BUNDESSTR. 55, D – 20146 HAMBURG

Degradable networks with general up and down time distributions

Cornelia Sauer and Hans Daduna

University of Hamburg, Department of Mathematics
Center of Mathematical Statistics and Stochastic Processes
Bundesstrasse 55, 20146 Hamburg

June 2004

Abstract:
We incorporate unreliable servers into the classical Jackson network models. The up and down times of the stations are nonexponential and additionally influenced by the global state of the network. This allows to consider versatile working period and repair time distributions. Our main result is a product form theorem for the joint steady state distribution of the combined queue length and reliability status of the network.

MSC (2000) subject classification: 60 K 25, 90 B 25, 60 K 37
Keywords: Performability, reliability, Jackson networks, blocking principles, product form solution, steady states

1 Introduction

The interplay of performance analysis and reliability analysis in complex networks has become an area of intensive research because of the need to understand the behaviour of modern communication networks and to control the functioning of the systems. Especially with respect to mobile networks reliability is becoming more and more important for the network architectures because provisioning quality of service, including high reliability for the customers, has to be guaranteed on the basis of not necessarily high network node availability [24]. Aspects of network control and routing protocols are dealt with in [16], [13] for the case of link failures of IP networks, and [10] for occurance of catastrophic events in mobile cellular networks.

There is a different but parallel line of research concerning problems of reliability and service: Queues with server breakdowns, for reviews see [25][Sect 3.3], or the introduction of [18].

An answer to parts of the questions that arise with respect to combined reliability and performance assessment is given by performability theory. This theory was developed to answer the needs of fault tolerant computer and communications systems. Surveys are e.g. [25], [8], [6].
The technical difficulties that arise with performability are due to having no product form results similar to that known in the area of performance analysis, connected with the works of Jackson [9], and Baskett, Chandy, Muntz, and Palacios [2].

Because of this either direct numerical methods are applied (Markov reward chains, etc.) or decomposition of the performance aspects on one side and reliability aspects on the other side. This is then combined with iterated procedures where the parameters of one model are inserted into the computations for the other model. The most prominent procedure is the reduced workload approximation, where the servers' availability is computed in a pure reliability model, and then in a pure queueing network model the stations are assumed to work with a rate adjusted by the availability factor. For a recent survey of this and related methods see [3]. In [15] a product form approximation is developed to obtain quantitative performance measures of degradable networks by using the property of near-complete decomposability and the consequences thereof.

Our approach is different, because we present an integrated performability model in the sense of [25]: "...the performance and reliability models are combined in this technique to yield a single monolithic performability model." Our main result is to incorporate into classical Jackson network models with stochastic routing and state-dependent service (Section 2) rather detailed and very versatile breakdown and repair processes (Section 3.1), and still retaining the product form steady state results (4). This is due to selecting suitable rerouting schemes for customers who arrive at nodes under repair (3.2): It turns out that we can reuse (with suitable adjustments) certain rerouting schemes that are used for the resolution of blocking in classical networks with blocking due to buffer overflow. In Section 5 we derive availability and performance measures for degradable networks and discuss in Section 6 the conclusions drawn from our results.

Our contribution in this paper is to open exponential networks. Closed Gordon-Newell networks with unreliable servers with exponential working and repair times are dealt with in [18]. Unreliable BCMP networks are investigated in [19]. The case of nonexponential breakdown and repair processes in these frameworks is part of our ongoing research.

Throughout the paper we set empty sums to 0, empty products to 1, $\frac{0}{0} := 0$. $\subset$ means 'subset or equal'.

2 Standard exponential networks

The starting point for our development are the open networks of single server nodes with state dependent service intensities, which were introduced by Jackson [9]. The relevant definition is:

Definition 2.1 (Jackson network) *A Jackson network is a network of service stations (nodes) numbered $\{1, 2, \dots, J\} := \bar{J}$. Station j is a single server with infinite waiting room under FCFS regime. Customers in the network are indistinguishable. At node j there is an external Poisson–λ_j arrival stream, $\lambda_j \geq 0$. Customers arriving at node j from the outside or from inside of the network request for an amount of service time there which is exponentially distributed with mean 1. Service at node j is provided with intensity $\mu_j(n_j) > 0$, if there are $n_j > 0$ customers present at node j. All service and interarrival times are assumed to be independent.*

Movements of the customers in the network are governed by a Markovian routing mechanism: A customer on leaving node i selects with probability $r(i,j) \geq 0$ to visit node j next, and then enters node j immediately, commencing service if he finds the server idle, otherwise he joins the tail of the queue at node j; with probability $r(i,0) \geq 0$ this customer decides to leave the network immediately ($\sum_{j=0}^{J} r(i,j) = 1$ for all $i \in \bar{J}$.) Given the departure node i the customer's routing decision is made independently of the network's history.

Let $\bar{J}_0 := \{0, 1, \dots, J\}$. With $\lambda := \sum_{j=1}^{J} \lambda_j$, $r(0,j) := \frac{\lambda_j}{\lambda}$ and $r(0,0) := 0$ we assume that the matrix $R := (r(i,j) : i, j \in \bar{J}_0)$ is irreducible.

Let $X_j(t)$ denote the number of customers present at node j at time $t \geq 0$, either waiting or in service (local queue length at node j), then $X(t) := (X_j(t) : j = 1, \dots, J)$ is the joint queue length vector of the network at time t and $X = (X(t) : t \geq 0)$ is the joint queue length process of the Jackson network with state space $E := \mathbb{N}^J$.

Theorem 2.2 ([9]) *The joint queue length process X of the Jackson network is a Markov process with transition rates $(q(x,y) : x, y \in E, x \neq y)$, which are for $x = (n_1, \dots, n_J) \in E$, $i, j \in \bar{J}, i \neq j$,*

$$\begin{aligned} q(n_1,\dots,n_i,\dots,n_J;\, n_1,\dots,n_i+1,\dots,n_J) &= \lambda_i, \\ \textit{and if} \quad n_i>0: \quad q(n_1,\dots,n_i,\dots,n_J;\, n_1,\dots,n_i-1,\dots,n_J) &= \mu_i(n_i)\, r(i,0), \\ q(n_1,\dots,n_i,\dots,n_j,\dots,n_J;\, n_1,\dots,n_i-1,\dots,n_j+1,\dots,n_J) &= \mu_i(n_i)\, r(i,j). \\ q(x,x) = -\sum_{y\in E\setminus\{x\}} q(x,y) \ \textit{and} \quad q(x,y)=0 & \qquad \textit{otherwise.} \end{aligned}$$

The traffic equation of the network for the flow rates into the nodes

$$\eta_j = \lambda_j + \sum_{i=1}^{J} \eta_i r(i,j), \quad j=1,\dots,J, \tag{1}$$

has a unique solution which we denote by $\eta = (\eta_1,\dots,\eta_J)$.
We assume henceforth X *to be ergodic. The unique stationary and limiting distribution* $\hat{\pi}$ *of* X *is*

$$\hat{\pi}(n_1,\dots,n_J) = K^{-1}\prod_{j=1}^{J}\prod_{k=1}^{n_j}\left(\frac{\eta_j}{\mu_j(k)}\right), \quad \textit{for } (n_1,\dots,n_J)\in I\!N^J$$

with normalization constant K. $\quad X$ *is ergodic if and only if* $K<\infty$.

Remark 2.3 *The class of exponential FCFS single servers with state dependent service rates encompasses the classical exponential FCFS multiservers with state independent service rates.*

3 The degradable network model

For the standard Jackson network described in Section 2 we consider the servers at the nodes to be unreliable and subject to random breakdowns, i.e. their operative periods are interrupted from time to time and the servers then have to be repaired. In their repair or down periods the nodes involved neither accept new customers nor continue to serve the present ones which will wait for the servers' return. Then the network's performance is degraded because a subset of nodes in the network is not available and the system is destabilized.

3.1 Control of breakdowns and repairs

The up and down periods for the nodes in the network obey special phase-type distributions, which are finite mixtures of Erlangian distributions. This class is dense with respect to the topology of weak convergence of probability measures in the set of all distributions on $(\mathbb{R}_+,\mathbb{B}_+)$, so that we can approximate any working and repair time distribution on $(\mathbb{R}_+,\mathbb{B}_+)$ sufficiently well. Fur further details see [20][Section I.6]. For $\kappa\in\mathbb{N}_+$ and $\gamma>0$ let us denote by

$$\Gamma_{\gamma,\kappa}(s) = 1 - e^{-\gamma s}\sum_{\iota=0}^{\kappa-1}\frac{(\gamma s)^\iota}{\iota!},\ s\geq 0,$$

the cumulative distribution function of the $\Gamma-$distribution with parameters γ and κ. The integer κ is positive and serves as phase-parameter for the number of independent exponential phases, each with mean γ^{-1}, the sum of which constitutes a random variable with distribution $\Gamma_{\gamma,\kappa}$ ($\Gamma_{\gamma,\kappa}$ is called a κ-stage Erlangian distribution with shape parameter γ.).

We define for each node $j \in \bar{J}$ for $a_j, b_j \in (0,\infty), M_j, N_j \in \mathbb{N}_+$ and probability functions $s_j(\cdot), t_j(\cdot)$ on $\{1,2,\ldots,M_j\}$ and $\{1,2,\ldots,N_j\}$ respectively two cumulative phase-type distribution functions on $\mathbb{R}_+$:

$$F_j(t) = \sum_{\kappa=1}^{M_j} s_j(\kappa)\Gamma_{a_j,\kappa}(t) \quad \text{and} \quad G_j(t) = \sum_{\kappa=1}^{N_j} t_j(\kappa)\Gamma_{b_j,\kappa}(t) \quad \text{respectively,} \tag{2}$$

which control the lengths of its up and down periods respectively. We denote the mean of these distributions by

$$EF_j = \sum_{\kappa=1}^{M_j} s_j(\kappa)\,\frac{\kappa}{a_j} =: \frac{1}{\alpha_j} \quad \text{and} \quad EG_j = \sum_{\kappa=1}^{N_j} t_j(\kappa)\,\frac{\kappa}{b_j} =: \frac{1}{\beta_j} \quad \text{respectively.}$$

The occurrence of breakdowns depends on the actual status of the network and on the interaction between the service stations: If $\bar{I} \subset \bar{J}$ is the set of nodes under repair then for node $j \in \bar{J}\backslash\bar{I}$ in up status the breakdown rate is $\Phi_j^{\bar{I}}(n_i : i \in \bar{I}\cup\{j\})$. For node $k \in \bar{I}$ the repair rate is $\Psi_k^{\bar{I}}(n_i : i \in \bar{I})$. These rates enable flexible modelling and control of the network by accelerating respective decelerating the occurrence of breakdowns and the repair.
The intensities $\Phi_j^{\bar{I}}(n_i : i \in \bar{I}\cup\{j\})$ and $\Psi_k^{\bar{I}}(n_i : i \in \bar{I})$ are set under some constraints. Multi-dimensional birth and death processes, see [22], suggest a rather general class by setting

Definition 3.1 (Control intensities) *The control intensities for $\bar{I} \subset \bar{J}$ being in down status are*

$$\Phi_j^{\bar{I}}(n_i : i \in \bar{I}\cup\{j\}) := \frac{\phi(n_i : i \in \bar{I}\cup\{j\})}{\phi(n_i : i \in \bar{I})}, \quad \text{and} \quad \Psi_k^{\bar{I}}(n_i : i \in \bar{I}) := \frac{\psi(n_i : i \in \bar{I})}{\psi(n_i : i \in \bar{I}\backslash\{k\})}, \tag{3}$$

with arbitrary nonnegative functions $\phi, \psi : \bigcup_{l\in\bar{J}_0} \mathbb{N}^l \to \mathbb{R}_+ = [0,\infty)$.
For standardization we set $\phi(n_i : i \in \emptyset) := 1 =: \psi(n_i : i \in \emptyset)$.

3.2 Rerouting mechanisms

When a node breaks down an ongoing service there is immediately stopped and only resumed when the node is repaired again. Customers are not allowed to enter nodes which are in down status so that we have to deal with a situation like blocking. Therefore the question is how we can restrict the customers' routing matrix to the set of nodes in up status in a suitable way keeping in mind that we pursue the objective of obtaining easy-to-handle product form results for the corresponding enlarged network process?
We describe three regimes to handle routing connected with nodes in down status. The rules originate from communication or production theory where they are used to resolve blocking situations. Blocking mechanisms have been developed for stochastic networks, in which a customer cannot leave a node after having obtained his service there because the population at his next destination node has reached the capacity of waiting plus service places there. Therefore the customer cannot join this node and the flow through the network according to the routing matrix is delayed or totally stopped. For detailed reviews of blocking techniques [17] or [1].

3.2.1 Stalling

The first principle to handle breakdowns is a global version of the Blocking Before Service-principle, see e.g. [12], [14], [4](Subsection 3.6.2), called Stalling in [21].

Definition 3.2 (STALLING) *If there is a breakdown of either a single node or a group of nodes, then all arrival streams to the network and all service processes at the nodes in up status are completely interrupted and resumed only after all failed nodes are repaired.*

We assume that servers in up status in a stalling situation behave as in a *hot stand–by* mode: These nodes can break down although working capacity is only hold ready and not effectively used. The breakdown rates are the same as if they are working on the frozen population size there.

3.2.2 Repeated service -random destination (RS-RD) Blocking

This principle originates from the theory of information blocking: A customer being served at node i chooses his next destination node j according to the routing table. If he cannot join node j the customer stays at node i to obtain another service. When this additional service expires the customer selects his destination node anew according to his routing instruction. This leads to:

Definition 3.3 (RS-RD with reversible routing) *Assume the nodes in $\bar{I} \subset \bar{J}$ are presently under repair. Then the routing is restricted to nodes with indices from $\bar{J}_0 \backslash \bar{I}$ according to*

$$\tilde{r}^{\bar{I}}(i,j) = \begin{cases} r(i,j), & i,j \in \bar{J}_0 \backslash \bar{I},\ i \neq j, \\ r(i,i) + \sum_{k \in \bar{I}} r(i,k), & i \in \bar{J}_0 \backslash \bar{I},\ i = j. \end{cases} \tag{4}$$

It follows for the external arrival rates

$$\tilde{\lambda}_j^{\bar{I}} = \lambda_j \text{ for nodes } j \in \bar{J} \backslash \bar{I} \text{ in up status}, \quad \tilde{\lambda}_j^{\bar{I}} = 0 \text{ for nodes } j \in \bar{I} \text{ under repair.}$$

Note that external arrivals may be rejected with positive probability:

$$\tilde{r}^{\bar{I}}(0,0) = r(0,0) + \sum_{k \in \bar{I}} r(0,k) = \sum_{k \in \bar{I}} \frac{\lambda_k}{\lambda} \geq 0.$$

We will apply RS-RD only when the original routing matrix, Definition 2.1, is reversible:

$$\eta_j \cdot r(j,i) = \eta_i \cdot r(i,j) \text{ for } i,j \in \bar{J}_0, \text{ with } \eta_j \text{ from (1).} \tag{5}$$

We cannot hope to remove assumption (5) without destroying our main results. Standard information blocking can be applied only to networks with (5) when explicit steady states are required.

3.2.3 Skipping

The third principle to handle breakdowns relies on methods from the theory of taboo probabilities for Markov chains. These principles seem to be introduced into the realm of queueing network theory independently several times and were used to resolve blocking, see e.g.[5] and [23][Chapter 3.6], (where it is called blocking and rerouting) and the references therein. As a general methodology SKIPPING was introduced by Schassberger [21].

Graphically spoken, the skipping rule for the customers' jumps under this restriction is in case of nodes in $\bar{I}$ being under repair:
If the customer present at node $i \in \bar{J}_0 \backslash \bar{I}$ selects (with probability $r(i,j)$) for his jump's destination node $j \in \bar{J}_0 \backslash \bar{I}$, this jump is allowed and immediately performed and the customer joins node j for service. If (with probability $r(i,k)$) he selects for his jump's destination node $k \in \bar{I}$, he only performs an imaginary jump to that node, spending no time there, but jumping on immediately according to the routing matrix $\mathcal{R}$, i.e. with probability $r(k,l)$ he selects another possible successor node l; if $l \in \bar{J}_0 \backslash \bar{I}$ then the jump is performed and the customer joins node l for service, but if $l \in \bar{I}$ the customer has to perform another random choice as if he would depart from l; and so on.

Definition 3.4 (SKIPPING) *Assume the nodes in $\bar{I}$ are the nodes of the Jackson network under repair. Then the skipping principle yields new Markov routing matrices $\hat{\mathcal{R}}^{\bar{I}} = \left(\hat{r}^{\bar{I}}(i,j) : i,j \in \bar{J}_0 \backslash \bar{I}\right)$ which are determined uniquely by the following set of equations:*

$$\hat{r}^{\bar{I}}(j,k) = r(j,k) + \sum_{i \in \bar{I}} r(j,i)\, \hat{r}^{\bar{I}}(i,k), \quad j \in \bar{J}_0,\ k \in \bar{J}_0 \backslash \bar{I}, \tag{6}$$

It follows for the new arrival rates $\hat{\lambda}_j^{\bar{I}}$ for $j \in \bar{J}$:

$$\hat{\lambda}_j^{\bar{I}} = \lambda \hat{r}^{\bar{I}}(0,j) = \lambda_j + \sum_{i \in \bar{I}} \lambda_i \hat{r}^{\bar{I}}(i,j) \text{ for } j \in \bar{J} \backslash \bar{I} \quad \text{and} \quad \hat{\lambda}_k^{\bar{I}} = 0 \text{ for } k \in \bar{I}. \tag{7}$$

Remark 3.5 *The solution of* (6) *can be computed explicitly as a matrix series.*
Skipping of blocked customers would also be obtained by setting the service intensities for new arrivals at the nodes under repair to ∞, and by applying LCFS regime.

4 The steady state results

For describing the degradable network's evolution we we attach to the state space $E = \mathbb{N}^J$ for the queue lengths process X additional processes Y, Z which carry information on the availability (reliability) of the system. Y is the set of nodes under repair, Z counts the residual (exponentially distributed) phases of the on-going up respective down periods of the respective nodes. The meaning of a prototype state

$$(\bar{I};\ n_1, \ldots, n_J; k_1, \ldots, k_J) \in \mathcal{P}(\bar{J}) \times \mathbb{N}^J \times \mathbb{N}^J$$

is: $\bar{I}$ is the set of nodes in repair. $n_i \in \mathbb{N}$ is the number of customers present at node $i \in \bar{J}$. If $i \in \bar{I}$, then $k_i \leq N_i$ is the residual number of exponential-b_i distributed repair phases. If $i \in \bar{J} \backslash \bar{I}$, then $k_i \leq M_i$ is the residual number of exponential-a_i distributed phases until the next breakdown of i. Collecting all feasible states we define for the degradable network a new Markov process $\widetilde{X} = (Y, X, Z)$ on the enlarged state space $\widetilde{E} \subset \mathcal{P}(\bar{J}) \times \mathbb{N}^J \times \mathbb{N}^J$. This results in

Theorem 4.1 *Let the supplemented state process $\widetilde{X}$ of a degradable Jackson network with breakdown and repair control from Section 3.1, and rerouting according to either* STALLING *(Definition 3.2), or* RS-RD-BLOCKING *(Definition 3.3 - with original reversible routing) or* SKIPPING *(Definition 3.4) be ergodic on $\widetilde{E}$. Then $\widetilde{X}$ has a stationary distribution of product form given by:*

For $(\emptyset; n_1, \ldots, n_J; k_1, \ldots, k_J) \in \widetilde{E}$:

$$\pi(\emptyset; n_1, \ldots, n_J; k_1, \ldots, k_J) = C^{-1} \prod_{j=1}^{J} \prod_{i=1}^{n_j} \left(\frac{\eta_j}{\mu_j(i)}\right) \prod_{j \in \bar{J}} \left(\frac{1}{a_j} \sum_{l=k_j}^{M_j} s_j(l)\right) \tag{8}$$

and for $(\bar{I};\ n_1, \ldots, n_J; k_1, \ldots, k_J) \in \widetilde{E}$ with $\bar{I} \neq \emptyset$:

$$\pi(\bar{I}; n_1, \ldots, n_J; k_1, \ldots, k_J) = C^{-1} \frac{\phi(n_i : i \in \bar{I})}{\psi(n_i : i \in \bar{I})} \prod_{j=1}^{J} \prod_{i=1}^{n_j} \left(\frac{\eta_j}{\mu_j(i)}\right) \prod_{j \in \bar{J} \backslash \bar{I}} \left(\frac{1}{a_j} \sum_{l=k_j}^{M_j} s_j(l)\right) \prod_{j \in \bar{I}} \left(\frac{1}{b_j} \sum_{l=k_j}^{N_j} t_j(l)\right). \tag{9}$$

Here $\eta = (\eta_1, \ldots, \eta_J)$ is the solution of the traffic equation (1) of the standard Jackson network, and C is the norming constant.

$\widetilde{X}$ *is ergodic if and only if* C *is finite, where* $C =$

$$\sum_{(n_1,\ldots,n_J)\in E}\left[\prod_{j=1}^{J}\prod_{i=1}^{n_j}\left(\frac{\eta_j}{\mu_j(i)}\right)\left(\sum_{\bar{I}\subset\bar{J}}\frac{\phi(n_i:i\in\bar{I})}{\psi(n_i:i\in\bar{I})}\sum_{(k_1,\ldots,k_J)\in\mathbb{N}^J}\prod_{j\in\bar{J}\setminus\bar{I}}\left(\frac{1}{a_j}\sum_{l=k_j}^{M_j}s_j(l)\right)\prod_{j\in\bar{I}}\left(\frac{1}{b_j}\sum_{l=k_j}^{N_j}t_j(l)\right)\right)\right]$$

$$=\sum_{(n_1,\ldots,n_J)\in E}\left[\prod_{j=1}^{J}\prod_{i=1}^{n_j}\left(\frac{\eta_j}{\mu_j(i)}\right)\left(\sum_{\bar{I}\subset\bar{J}}\frac{\phi(n_i:i\in\bar{I})}{\psi(n_i:i\in\bar{I})}\prod_{j\in\bar{J}\setminus\bar{I}}\left(\frac{1}{\alpha_j}\right)\prod_{j\in\bar{I}}\left(\frac{1}{\beta_j}\right)\right)\right].$$

Proof: We carry out the proof in case of rerouting according to RS-RD-Blocking. The proofs in case of Stalling or Skipping proceed analogously.

The steady state equations for the network processes are:

$$\begin{aligned}
&\pi(\emptyset;n_1,\ldots,n_J;k_1,\ldots,k_J)\left(\sum_{j=1}^{J}(1-\delta_{0n_j})\mu_j(n_j)(1-r(j,j))+\sum_{j=1}^{J}\lambda_j+\sum_{j=1}^{J}a_j\,\Phi_j^{\emptyset}(n_j)\right)\\
&=\sum_{j=1}^{J}(1-\delta_{0n_j})\pi(\emptyset;n_1,\ldots,n_j-1,\ldots,n_J;k_1,\ldots,k_J)\lambda_j\\
&+\sum_{j=1}^{J}(1-\delta_{0n_j})\sum_{\substack{i=1\\ i\neq j}}^{J}\pi(\emptyset;n_1,\ldots,n_i+1,\ldots,n_j-1,\ldots,n_J;k_1,\ldots,k_J)\mu_i(n_i+1)r(i,j)\\
&+\sum_{j=1}^{J}\pi(\emptyset;n_1,\ldots,n_j+1,\ldots,n_J;k_1,\ldots,k_J)\mu_j(n_j+1)r(j,0)\\
&+\sum_{j=1}^{J}\pi(\emptyset;n_1,\ldots,n_J;k_1,\ldots,k_j+1,\ldots,k_J)\,a_j\,(1-\delta_{k_jM_j})\,\Phi_j^{\emptyset}(n_j)\\
&+\sum_{j=1}^{J}\pi(\{j\};n_1,\ldots,n_J;k_1,\ldots,k_{j-1},1,k_{j+1},\ldots,k_J)\,b_j\,\Psi_j^{\{j\}}(n_j)\,s_j(k_j) \qquad (10)
\end{aligned}$$

for all $(\emptyset;n_1\ldots,n_J;k_1,\ldots,k_J)\in\{\emptyset\}\times E,$

For all $(\bar I;\, n_1,\dots,n_J;k_1,\dots,k_j)\in\widetilde E$ with $\bar I\neq\emptyset$ we have

$$\pi(\bar I;\, n_1,\dots,n_J;k_1,\dots,k_J)\Bigg(\sum_{j\in\bar J\backslash\bar I}(1-\delta_{0n_j})\mu_j(n_j)(1-\tilde r^{\bar I}(j,j))+\sum_{j\in\bar J\backslash\bar I}\tilde\lambda_j^{\bar I}$$

$$+\sum_{j\in\bar J\backslash\bar I}a_j\,\Phi_j^{\bar I}(n_i:i\in\bar I\cup\{j\})+\sum_{j\in\bar I}b_j\,\Psi_j^{\bar I}(n_i:i\in\bar I)\Bigg)$$

$$=\sum_{j\in\bar J\backslash\bar I}(1-\delta_{0n_j})\pi(\bar I;\, n_1,n_2,\dots,n_j-1,\dots,n_J;k_1,\dots,k_J)\tilde\lambda_j^{\bar I}$$

$$+\sum_{j\in\bar J\backslash\bar I}(1-\delta_{0n_j})\sum_{\substack{k\in\bar J\backslash\bar I\\ k\neq j}}\pi(\bar I;\, n_1,\dots,n_k+1,\dots,n_j-1,\dots,n_J;k_1,\dots,k_J)\mu_k(n_k+1)\tilde r^{\bar I}(k,j)$$

$$+\sum_{j\in\bar J\backslash\bar I}\pi(\bar I;\, n_1,n_2,\dots,n_j+1,\dots,n_J;k_1,\dots,k_J)\mu_j(n_j+1)\tilde r^{\bar I}(j,0)$$

$$+\sum_{j\in\bar J\backslash\bar I}\pi(\bar I;n_1,\dots,n_J;k_1,\dots,k_j+1,\dots,k_J)\,a_j\,(1-\delta_{k_jM_j})\,\Phi_j^{\bar I}(n_i:i\in\bar I\cup\{j\})$$

$$+\sum_{j\in\bar I}\pi(\bar I;n_1,\dots,n_J;k_1,\dots,k_j+1,\dots,k_J)\,b_j\,(1-\delta_{k_jN_j})\,\Psi_j^{\bar I}(n_i:i\in\bar I)$$

$$+\sum_{j\in\bar J\backslash\bar I}\pi(\bar I\cup\{j\};n_1,\dots,n_J;k_1,\dots,k_{j-1},1,k_{j+1},\dots,k_J)\,b_j\,\Psi_j^{\bar I\cup\{j\}}(n_i:i\in\bar I\cup\{j\})\,s_j(k_j)$$

$$+\sum_{j\in\bar I}\pi(\bar I\backslash\{j\};n_1,\dots,n_J;k_1,\dots,k_{j-1},1,k_{j+1},\dots,k_J)\,a_j\,\Phi_j^{\bar I\backslash\{j\}}(n_i:i\in\bar I)\,t_j(k_j). \tag{11}$$

We have to show that the distribution given in (8) and (9) solves the equations (10) and (11).
In the first equation (10) for each $j\in\bar J$ the term $\pi(\emptyset;n_1,\dots,n_J;k_1,\dots,k_J)\,a_j\,\Phi_j^{\emptyset}(n_j)$ on the left hand side is equal to the following term on the right hand side

$$\pi(\emptyset;n_1,\dots,n_J;k_1,\dots,k_j+1,\dots,k_J)\,a_j\,(1-\delta_{k_jM_j})\,\Phi_j^{\emptyset}(n_j)$$

$$+\pi(\{j\};n_1,\dots,n_J;k_1,\dots,k_{j-1},1,k_{j+1},\dots,k_J)\,b_j\,\Psi_j^{\{j\}}(n_j)\,s_j(k_j)$$

$$=\pi(\emptyset;n_1,\dots,n_J;k_1,\dots,k_j\dots,k_J)\,\phi(n_j)\left(\sum_{l=k_j}^{M_j}s_j(l)\right)^{-1}\left(a_j\,(1-\delta_{k_jM_j})\left(\sum_{l=k_j+1}^{M_j}s_j(l)\right)+a_j\,s_j(k_j)\right)$$

$$=\pi(\emptyset;n_1,\dots,n_J;k_1,\dots,k_J)\,a_j\,\Phi_j^{\emptyset}(n_j).$$

Thus, the global stationary equation of the standard Jackson network with J nodes is left over. It is solved by the distribution expression (8) when considering the expression $\prod_{j=1}^J\left(\frac{1}{a_j}\sum_{l=k_j}^{M_j}s_j(l)\right)$ as a constant.
In the second equation (11) for any $j\in\bar J\backslash\bar I$ the term on the left hand side

$$\pi(\bar I;n_1,\dots,n_J;k_1,\dots,k_J)\,a_j\,\Phi_j^{\bar I}(n_i:i\in\bar I\cup\{j\})$$

is equal to the following term on the right hand side

$$\pi(\bar{I};n_1,\dots,n_J;k_1,\dots,k_j+1,\dots,k_J)\,a_j\,(1-\delta_{k_jM_j})\,\Phi_j^{\bar{I}}(n_i:i\in\bar{I}\cup\{j\})$$
$$+\pi(\bar{I}\cup\{j\};n_1,\dots,n_J;k_1,\dots,k_{j-1},1,k_{j+1},\dots,k_J)\,b_j\,\Psi_j^{\bar{I}\cup\{j\}}(n_i:i\in\bar{I}\cup\{j\})\,s_j(k_j)$$
$$=\pi(\bar{I};n_1,\dots,n_J;k_1,\dots,k_j\dots,k_J)\Phi_j^{\bar{I}}(n_i:i\in\bar{I}\cup\{j\})\Big(\sum_{l=k_j}^{M_j}s_j(l)\Big)^{-1}\left(a_j\,(1-\delta_{k_jM_j})\left(\sum_{l=k_j+1}^{M_j}s_j(l)\right)+a_j\,s_j(k_j)\right)$$
$$=\pi(\bar{I};n_1,\dots,n_J;k_1,\dots,k_J)\,a_j\,\Phi_j^{\bar{I}}(n_i:i\in\bar{I}\cup\{j\}).$$

Moreover for each $j\in\bar{I}$ the term $\pi(\bar{I};n_1,\dots,n_J;k_1,\dots,k_J)\,b_j\,\Psi_j^{\bar{I}}(n_i:i\in\bar{I})$ on the left hand side is equal to the following term on the right hand side

$$\pi(\bar{I};n_1,\dots,n_J;k_1,\dots,k_j+1,\dots,k_J)\,b_j\,(1-\delta_{k_jN_j})\,\Psi_j^{\bar{I}}(n_i:i\in\bar{I})$$
$$+\pi(\bar{I}\backslash\{j\};n_1,\dots,n_J;k_1,\dots,k_{j-1},1,k_{j+1},\dots,k_J)\,a_j\,\Phi_j^{\bar{I}\backslash\{j\}}(n_i:i\in\bar{I})\,t_j(k_j)$$
$$=\pi(\bar{I};n_1,\dots,n_J;k_1,\dots,k_j\dots,k_J)\Psi_j^{\bar{I}}(n_i:i\in\bar{I})\Big(\sum_{l=k_j}^{N_j}t_j(l)\Big)^{-1}\left(b_j\,(1-\delta_{k_jN_j})\left(\sum_{l=k_j+1}^{N_j}t_j(l)\right)+b_j\,t_j(k_j)\right)$$
$$=\pi(\bar{I};n_1,\dots,n_J;k_1,\dots,k_J)\,b_j\,\Psi_j^{\bar{I}}(n_i:i\in\bar{I}).$$

After cancelling all these terms the remainder is the global steady state equation of the standard Jackson network with node set $\bar{J}\backslash\bar{I}$, service rates $\mu_j(\cdot)$, external arrival rates $\tilde{\lambda}_j^{\bar{I}}$ and routing probabilities $\tilde{r}^{\bar{I}}(i,j)$ for $i,j\in\bar{J}\backslash\bar{I}$ which is solved by

$$\pi(\bar{I};n_1,\dots,n_J;k_1,\dots,k_J)=\prod_{j\in\bar{J}\backslash\bar{I}}\prod_{i=1}^{n_j}\left(\frac{\tilde{\eta}_j^{\bar{I}}}{\mu_j(i)}\right)\tilde{C}^{-1}(n_i:i\in\bar{I},k_i:i\in\bar{J}).$$

where n_i, $i\in\bar{I}$, and k_i, $i\in\bar{J}$, are constants and $\tilde{\eta}^{\bar{I}}=(\tilde{\eta}_j^{\bar{I}}:j\in\bar{J}\backslash\bar{I})$ is the solution of the traffic equation

$$\tilde{\eta}_j^{\bar{I}}=\tilde{\lambda}_j^{\bar{I}}+\sum_{i\in\bar{J}\backslash\bar{I}}\tilde{\eta}_i^{\bar{I}}\tilde{r}^{\bar{I}}(i,j),\quad j\in\bar{J}\backslash\bar{I}.$$

Therefore it is sufficient to show that $\tilde{\eta}_j^{\bar{I}}=\eta_j$ for all $j\in\bar{J}\backslash\bar{I}$ holds. Using the definition of the new routing probabilities $\tilde{r}^{\bar{I}}(i,j)$ from (4) this identity can be verified:

$$\tilde{\lambda}_j^{\bar{I}}+\sum_{i\in\bar{J}\backslash\bar{I}}\eta_i\tilde{r}^{\bar{I}}(i,j)=\lambda_j+\sum_{\substack{i\in\bar{J}\backslash\bar{I}\\ i\neq j}}\eta_ir(i,j)+\eta_j\Big(r(j,j)+\sum_{i\in\bar{I}}r(j,i)\Big)$$
$$=\lambda_j+\sum_{i\in\bar{J}\backslash\bar{I}}\eta_ir(i,j)+\sum_{i\in\bar{I}}\eta_jr(j,i)=\eta_j-\sum_{i\in\bar{I}}\eta_ir(i,j)+\sum_{i\in\bar{I}}\eta_ir(i,j)=\eta_j,\ j\in\bar{J}\backslash\bar{I},$$

where the last but one equality follows from reversibility. Finally we define

$$\tilde{C}^{-1}(n_i:i\in\bar{I},k_i:i\in\bar{J})=C^{-1}\left(\prod_{j\in\bar{I}}\prod_{i=1}^{n_j}\frac{\eta_j}{\mu_j(i)}\right)\frac{\phi(n_i:i\in\bar{I})}{\psi(n_i:i\in\bar{I})}\Big(\prod_{j\in\bar{J}\backslash\bar{I}}\frac{1}{a_j}\sum_{l=k_j}^{M_j}s_j(l)\Big)\Big(\prod_{j\in\bar{I}}\frac{1}{b_j}\sum_{l=k_j}^{N_j}t_j(l)\Big)$$

to obtain the proposed form for $\pi(\bar{I};\,n_1,\dots,n_J;k_1,\dots,k_J)$. ■

Remark 4.2 *This product form structure of the steady state indicates that the computation of the norming constant C is in general not an easy procedure. Nevertheless we can apply algorithms* *similar to those developed in classical performance analysis for queueing networks - especially in case of closed networks. For open networks with state dependent rates we are faced with problems similar to those in Jackson network theory. As will be shown in Proposition 5.1 for special cases the computational burdenings can be reduced considerably because of the factorization* (13). *Note that in any case C depends on the up and down time distributions only through their means.*

5 Availability and performance measures

The network processes $\widetilde{X} = (Y, X, Z)$ include as observables the information on availability (reliability properties) and queue lengths (performance properties). It is obvious that our result in Theorem 4.1 which provides an explicit expression for the steady state distribution opens the path to study in depth the interplay and interdependence of the queueing and reliability processes. Moreover, from the very definition of the state space of $\widetilde{X}(t) = (Y(t), X(t), Z(t))$ we can determine reliability aspects and performance aspects simply by computing marginal distributions and their characteristics.
Due to the complexity of the modelling schemes which fall into our general class of systems with state dependent service rates, state dependent reair and breakdown rates, the performance measures computed should be still envolved - although explicitly at hand.
We discuss an example of sufficient complexity which on the other hand shows some striking simplifications in the formulae.

Proposition 5.1 *Consider degradable Jackson network in steady state with general operating and repair time distributions* (2) *and with customer-load independent accelerating and decelerating rates*

$$\phi(n_i : i \in \bar{H}) := \phi(\bar{H}), \quad \psi(n_i : i \in \bar{H}) := \psi(\bar{H}) \ \textit{for all } \bar{H} \subset \bar{J}, \bar{H} \neq \emptyset. \tag{12}$$

(a) *The probability $\pi(\bar{I})$ that exactly the nodes in set $\bar{I} \subset \bar{J}$ are under repair is given by*

$$\pi(\bar{I}) = \frac{\phi(\bar{I})}{\psi(\bar{I})} \left(\prod_{j \in \bar{J} \setminus \bar{I}} \frac{1}{\alpha_j} \right) \left(\prod_{j \in \bar{I}} \frac{1}{\beta_j} \right) \left(\sum_{\bar{K} \subset \bar{J}} \left(\prod_{j \in \bar{J} \setminus \bar{K}} \frac{1}{\alpha_j} \right) \left(\prod_{j \in \bar{K}} \frac{1}{\beta_j} \right) \frac{\phi(\bar{K})}{\psi(\bar{K})} \right)^{-1}.$$

(b) *Average queue lengths $E(X_k)$ are the same as in the standard Jackson network, $k \in \bar{J}$.*

Proof :(a) $\pi(\bar{I}) = \sum_{(n_1,\ldots,n_J) \in \mathbb{N}^J} \sum_{(k_1,\ldots,k_J) \in \mathbb{N}^J} \pi(\bar{I}; n_1, \ldots, n_J; k_1, \ldots, k_J)$

$$= C^{-1} \sum_{(n_1,\ldots,n_J) \in \mathbb{N}^J} \frac{\phi(\bar{I})}{\psi(\bar{I})} \prod_{j=1}^{J} \prod_{i=1}^{n_j} \left(\frac{\eta_j}{\mu_j(i)} \right) \sum_{(k_1,\ldots k_J) \in \mathbb{N}^J} \prod_{j \in \bar{J} \setminus \bar{I}} \left(\frac{1}{a_j} \sum_{l=k_j}^{M_j} s_j(l) \right) \prod_{j \in \bar{I}} \left(\frac{1}{b_j} \sum_{l=k_j}^{N_j} t_j(l) \right)$$

$$= C^{-1} K \frac{\phi(\bar{I})}{\psi(\bar{I})} \prod_{j \in \bar{J} \setminus \bar{I}} \left(\frac{1}{a_j} \sum_{k_j=1}^{M_j} \sum_{l=k_j}^{M_j} s_j(l) \right) \prod_{j \in \bar{I}} \left(\frac{1}{b_j} \sum_{k_j=1}^{N_j} \sum_{l=k_j}^{N_j} t_j(l) \right)$$

$$= C^{-1} K \frac{\phi(\bar{I})}{\psi(\bar{I})} \prod_{j \in \bar{J} \setminus \bar{I}} \left(\frac{1}{a_j} \sum_{l=1}^{M_j} l \cdot s_j(l) \right) \prod_{j \in \bar{I}} \left(\frac{1}{b_j} \sum_{l=1}^{N_j} l \cdot t_j(l) \right)$$

$$= \left(\sum_{\bar{K} \subset \bar{J}} \left(\prod_{j \in \bar{J} \setminus \bar{K}} \frac{1}{\alpha_j} \right) \left(\prod_{j \in \bar{K}} \frac{1}{\beta_j} \right) \frac{\phi(\bar{K})}{\psi(\bar{K})} \right)^{-1} \left(\prod_{j \in \bar{J} \setminus \bar{I}} \frac{1}{\alpha_j} \right) \left(\prod_{j \in \bar{I}} \frac{1}{\beta_j} \right) \frac{\phi(\bar{I})}{\psi(\bar{I})}.$$

(b) Let $\hat{E}(X_k)$ be the mean queue length at node $k \in \bar{J}$ in the stationary standard Jackson network of Theorem 2.2. Then

$$E(X_k) = \sum_{\bar{K}\subset\bar{J}} \sum_{(k_1,\ldots,k_J)\in\mathbb{N}^J} \sum_{(n_1,\ldots,n_J)\in\mathbb{N}^J} n_k \cdot \pi(\bar{K}; n_1,\ldots,n_J; k_1,\ldots,k_J)$$

$$= C^{-1} \sum_{(n_1,\ldots,n_J)\in\mathbb{N}^J} n_k \prod_{j=1}^{J}\prod_{i=1}^{n_j}\left(\frac{\eta_j}{\mu_j(i)}\right) \sum_{\bar{K}\subset\bar{J}} \frac{\phi(\bar{K})}{\psi(\bar{K})} \sum_{(k_1,\ldots k_J)\in\mathbb{N}^J} \prod_{j\in\bar{J}\setminus\bar{K}} \left(\frac{1}{a_j}\sum_{l=k_j}^{M_j} s_j(l)\right) \prod_{j\in\bar{K}} \left(\frac{1}{b_j}\sum_{l=k_j}^{N_j} t_j(l)\right)$$

$$= C^{-1}\, K\, \hat{E}(X_k) \sum_{\bar{K}\subset\bar{J}} \frac{\phi(\bar{K})}{\psi(\bar{K})} \left(\prod_{j\in\bar{J}\setminus\bar{K}} \frac{1}{\alpha_j}\right) \left(\prod_{j\in\bar{K}} \frac{1}{\beta_j}\right) = \hat{E}(X_k) \text{ for all } k \in \bar{J}. \qquad \blacksquare$$

Remark 5.2 *In case of rates* (12) *the breakdown processes and the repair can be thought to be external with regard to the service processes because the operating and repair times are independent of the arrivals and services. Nevertheless the processes X describing the service status and Y, Z describing the stations' availability are obviously strongly correlated. Therefore the following observation is surprising. By simple computations we see that the equilibrium distribution has the following remarkable product structure:*

$$\pi(\bar{I}; n_1,\ldots,n_J; k_1,\ldots,k_J) = \pi(\bar{I})\,\hat{\pi}(n_1,\ldots,n_J)\,\pi(k_1,\ldots,k_J) \text{ for } \bar{I}\subset\bar{J}, (n_1,\ldots,n_J)\in E. \quad (13)$$

with $\hat{\pi}(n_1,\ldots,n_J)$ as steady state distribution in the standard Jackson network and $\pi(k_1,\ldots,k_J)$ the marginal distribution for the residual repair, resp. operating time phases in the steady state of Theorem 4.1.
This is a structural analog to Jackson's celebrated Productform Theorem *[9]:* Asymptotically the queueing processes and the reliability processes decouple and behave as if they are independent. *Recall that $\hat{\pi}(n_1,\ldots,n_J)$ decouples further - the local queue lengths at the nodes behave asymptotically independent according to Jackson's theorem. And even more: $\pi(k_1,\ldots,k_J)$ decouples in a completely similar way.*
It follows that the mean value analysis for availability and performance measures which deals with steady state quantities can be decoupled as well. This is the rational behind the Proposition 5.1.

Moreover, the steady state distribution of the availability process Y is insensitive against variations of the shape of the up and down periods' distributions as long as the mean breakdown respective repair times are not changed. Thus, reliability analysis of the degradable network with respect to equilibrium availability distributions and quantities derived from these fundamental characteristics can be reduced to the case of exponentially distributed up and down periods with the correct means α_j^{-1} respective $\beta_j^{-1}, j \in \bar{J}$. This reduces the state space and the technical burdenings of the system under investigation considerably.

From a system's point of view the possibly most important performance measure is the steady state throughput. For open networks without performance reductions due to breakdowns this is simply λ, see Definition 2.1. By simple flow balancing arguments we can derive throughput results for the general case of open degradable networks as well.

Proposition 5.3 *The throughput TH in our degradable Jackson networks is the effective external arrival rate. This is*
(a) *for networks with rerouting under terms of* STALLING

$$TH(st) = \lambda \cdot \pi(\emptyset),$$

(b) *for networks with rerouting under terms of* RS-RD-BLOCKING

$$TH(rs\text{-}rd) = \sum_{\bar{I} \subset \bar{J}} \pi(\bar{I}) \cdot \sum_{j \in \bar{J} \setminus \bar{I}} \lambda_j,$$

(c) *for networks with rerouting under terms of* SKIPPING

$$TH(sk) = \sum_{\bar{I} \subset \bar{J}} \pi(\bar{I}) \left[\lambda - \sum_{i \in \bar{I}} \lambda_i \, \hat{r}^{\bar{I}}(i,0) \right],$$

where $\pi(\bar{I})$ *is the probability that exactly the nodes in set* $\bar{I} \subset \bar{J}$ *are under repair.*
We have $TH(st) < TH(rs\text{-}rd) < TH(sk) < \lambda$, *which is intuitively clear.*

6 Conclusions

We have obtained new product form stationary distributions for networks with degradable servers, where the working and repair periods have general distributions and their duration may depend on the network load. These formulae allow to study in depth availability and queueing behavior of the nodes and, more important, the interplay between these properties. The results provide information on the long time behavior and the steady state respectively of the network.

While planning such networks and while running the operating systems we are (somehow in contrast to this) interested that failures are transient or repaired quickly. This leads to investigations of networks operating in different modes of behavior: What does happen if failures occur? What is the network's performance given a specified failure set is in force? It turns out that our results open the way to describe these modes of conditional stochastic behavior as well, by assuming that the network evolves under suitably chosen PALM distributions. Further investigations of these aspects are the topic of a forthcoming paper.

Literatur

[1] Balsamo, S., De Nitto Persone, V. and Onvural, R. (2001) *Analysis of Queueing Networks with Blocking.* Kluwer academic publishers.

[2] Baskett, F., Chandy, M., Muntz, R. and Palacios, F.G. (1975) Open, closed and mixed networks of queues with different classes of customers. *Journal of the Association for Computing Machinery*, 22, 248-260.

[3] Chakka, R. and Mitrani, I. (1996) Approximate solutions for open networks with breakdowns and repairs. In F. P. Kelly, S. Zachary and I. Ziedins, editors, *Stochastic Networks - Theory and applications*, chapter 16, 267-280, Clarendon Press, Oxford.

[4] Dijk, N.M. Van (1993) *Queueing Networks and Product Forms - A Systems Approach.* Wiley, Chichester.

[5] Economou, A. and Fakinos, D. (1998) Product form stationary distributions for queueing networks with blocking and rerouting. *Queueing Systems and Their Applictions*, 30, 251 - 260.

[6] German, R., Luethi, J. and Telek, M. (2001), editors. *Proceedings of the Fifth International Workshop on Performability Modeling of Computer and Communication Systems,* September 15-16,2001, Erlangen.

[7] Gordon, W.J. and Newell, G.F. (1967) Closed queueing systems with exponential servers. *Operations Research*, 15, 254-265.

[8] Haverkort, B.R., Marie, R., Rubino, G. and Trivedi, K. (2001), editors, *Performability Modelling - Techniques and Tools.* Wiley.

[9] Jackson, J.R. (1957) Networks of waiting lines. *Operations Research*, 5, 518-521.

[10] Kordybach, K. and Nousiainen, S. Radio resource management in WCDMA-based networks in emergency situations In J. Charzinski, R. Lehnert, P. Tran-Gia, editors, *Providing QoS in Heterogeneous Environments (Proceedings of the 18th ITC)*, volume 5b of Teletraffic Science and Engineering, pp. 331-338, Elsevier Science, Amsterdam, 2003.

[11] Meyer, J.F. (1984) Performability modeling of distributed real-time systems. In G. Iazeolla, P.J. Courtois and A. Hordijk, editors, *Mathematical Computer Performance and Reliability*, 361-372, North-Holland, Amsterdam.

[12] Mikou, N. (1988) A two-node Jackson's network subject to breakdowns. *Stochastic Models*, 4, 523-552.

[13] Menth, M., Kopf, S. and Charzinski, J. Impact of resilience requirements on the performance of network admission control methods Technical Report No. 309, University of Würzburg, Institute of Computer Sciences

[14] Mitrani, I. (1974) Networks of unreliable computers. In E. Gelenbe and R. Mahl, editors, *Computer Architectures and Networks*, 359-374, North-Holland, Amsterdam.

[15] Müller-Clostermann, B. (1988) An approximate product form for a class of degradable queueing networks. *Performance Evaluation*, 8, 165-172.

[16] Nucci, A., Schroeder, B., Bhattacharyya, S., Taft, N., and Diot, C. IGP link weight assignment for transient link failures In J. Charzinski, R. Lehnert, P. Tran-Gia, editors, *Providing QoS in Heterogeneous Environments (Proceedings of the 18th ITC)*, volume 5a of Teletraffic Science and Engineering, pp. 321-330, Elsevier Science, Amsterdam, 2003.

[17] Perros, H.G. (1990) Approximation algorithms for open queueing networks with blocking. In H. Takagi, editor, *Stochastic Analysis of Computer and Communication Systems*, 451-498, North-Holland, Amsterdam.

[18] Sauer, C. and Daduna, H. (2003) Availability formulas and performance measures for separable degradable networks, *Economic Quality Control*, Vol.18, No.2, pp. 165-194, 2003.

[19] Sauer, C. and Daduna, H. (2003) Separable networks with unreliable servers. In J. Charzinski, R. Lehnert, P. Tran-Gia, editors, *Providing QoS in Heterogeneous Environments (Proceedings of the 18th ITC)*, volume 5b of Teletraffic Science and Engineering, pp. 821-830, Elsevier Science, Amsterdam, 2003.

[20] Schassberger, R. (1973) *Warteschlangen.* Springer, Wien.

[21] Schassberger, R. (1983) Decomposable stochastic networks: Some observations. In F. Baccelli and G. Fayolle, editors, *Modelling and Performance Evaluation Methodology*, volume 60 of *Lecture Notes in Control and Information Sciences*, chapter IV, 137-150, Springer, Berlin, 1984. Proceedings of the International Seminar, Paris, France, January 24-26.

[22] Serfozo, R.F. (1993) Queueing networks with dependent nodes and concurrent movements. *Queueing Systems*, 13, 143-182.

[23] Serfozo, R.F. (1999) *Introduction to Stochastic Networks.* Applications of Mathematics 44, Springer, New York.

[24] Schopp, M. and Viallon, J. Service availability in distributed network architectures: Evaluation and planning issues In J. Charzinski, R. Lehnert, P. Tran-Gia, editors, *Providing QoS in Heterogeneous Environments (Proceedings of the 18th ITC)*, volume 5a of Teletraffic Science and Engineering, pp. 251-260, Elsevier Science, Amsterdam, 2003.

[25] Trivedi, K.S., Malhotra, M. (1993) Reliability and performability techniques and tools: A survey. Preprint, Dept. of Electrical Engineering, Duke University, Durham.