# A universal model-free and safe adaptive cruise control mechanism

Thomas Berger and Anna-Lena Rauert

***Abstract*— We consider the problem of vehicle following, where a safety distance to the leader vehicle is guaranteed at all times and a favourite velocity is reached as far as possible. We introduce the funnel cruise controller as a novel universal adaptive cruise control mechanism which is model-free and achieves the aforementioned control objectives. The controller consists of a velocity funnel controller, which directly regulates the velocity when the leader vehicle is far away, and a distance funnel controller, which regulates the distance to the leader vehicle when it is close so that the safety distance is never violated. A sketch of the feasibility proof is given. The funnel cruise controller is illustrated by a simulation of three different scenarios which may occur in daily traffic.**

***Index Terms*— Autonomous vehicles, adaptive cruise control, nonlinear systems, funnel control, safety guarantees**
***AMS subject classifications*— 93C10, 93C40, 70Q05**

## I. INTRODUCTION

With traffic steadily increasing, simple cruise control (see e.g. [1]), which holds the velocity on a constant level, is becoming less useful. A controller which additionally allows a vehicle to follow the vehicle in front of it while continually adjusting speed to maintain a safe distance is a suitable alternative. Various methods which achieve this are available in the literature, see e.g. the survey [2] on adaptive cruise control systems. A common method is the use of proportional-integral-derivative (PID) controllers, see e.g. [1], [3], [4], [5], which however are not able to guarantee any safety.

Another popular method is model predictive control (MPC), where the control action is defined by repeated solution of a finite-horizon optimal control problem. A two-mode MPC controller is developed in [6], where the controller switches between velocity and distance control. The MPC method introduced in [7] incorporates the fuel consumption and driver desired response in the cost function of the optimal control problem. In [8] a method which guarantees both safety and comfort is developed. It consists of a nominal controller, which is based on MPC, and an emergency controller which takes over when MPC does not provide a safe solution.

Control methods based on control barrier functions which penalize the violation of given constraints have been developed in [9], [10]. While safety constraints are automatically guaranteed by this approach, it may be hard to find a suitable control barrier function. Another recent method is correct-by-construction adaptive cruise control [11], which is also able to guarantee safety. However, the computations are based on a so called finite abstraction of the system (which is already expensive) and changes of the system parameters require a complete re-computation of the finite abstraction.

Drawbacks of the aforementioned approaches are that either safety cannot be guaranteed (as in [3], [4], [5]) or the model must be known exactly (as in [9], [6], [8], [10], [11], [7]). However, the requirements on driver assistance systems are increasing steadily. It is expected that in the near future autonomous vehicles will completely take over all driving duties. Therefore, a cruise control mechanism is desired which achieves both: under any circumstances (in particular, in emergency situations) the prescribed safety distance to the preceding vehicle is guaranteed and at the same time the parameters of the model, such as aerodynamic drag or rolling friction, must not be known exactly, i.e., the control mechanism is model-free. The latter property also guarantees that the controller is inherently robust, in particular with respect to uncertainties, modelling errors or external disturbances. Another requirement on the controller is that it should be simple in its design and of low complexity, and that it only requires the measurement of the velocity and the distance to the leading vehicle. We stress that in a lot of other approaches as e.g. [6], [3], [8], [7] the position, velocity and/or acceleration of the leading vehicle must be known at each time.

In the present paper we propose a novel control design which satisfies the above requirements. Our control design is based on the funnel controller which was developed in [12], see also the survey [13] and the recent paper [14]. The funnel controller is an adaptive controller of high-gain type and has been successfully applied e.g. in temperature control of chemical reactor models [15], control of industrial servo-systems [16], [17], [18], DC-link power flow control [19], voltage and current control of electrical circuits [20], and control of peak inspiratory pressure [21].

In our design we will distinguish two different cases. If the preceding vehicle is far away, i.e., the distance to it is larger than the safety distance plus some constant, then a velocity funnel controller will be active which simply regulates the velocity of the vehicle to the desired pre-defined velocity. If the preceding vehicle is close, then a distance funnel controller will be active which regulates the distance to the preceding vehicle to stay within a predefined performance funnel in front of the safety distance. The combination of these two controllers results in a funnel cruise controller which guarantees safety at all times. We like to stress that

This work was supported by the German Research Foundation (Deutsche Forschungsgemeinschaft) via the grant BE 6263/1-1.
Thomas Berger and Anna-Lena Rauert are with the Fachbereich Mathematik, Universität Hamburg, Bundesstraße 55, 20146 Hamburg, Germany, thomas.berger@uni-hamburg.de, anna-lena.rauert@studium.uni-hamburg.de

the distance funnel controller does not directly regulate the position of the vehicle, but a certain weighting between position and velocity; hence, a relative degree one controller suffices.

### A. Nomenclature

| | |
|---|---|
| $\mathbb{R}_{\geq 0}$ | $= [0,\infty)$ |
| $\mathscr{L}^\infty_{\text{loc}}(I \to \mathbb{R}^n)$ | the set of locally essentially bounded functions $f: I \to \mathbb{R}^n$, $I \subseteq \mathbb{R}$ an interval |
| $\mathscr{L}^\infty(I \to \mathbb{R}^n)$ | the set of essentially bounded functions $f: I \to \mathbb{R}^n$ |
| $\|f\|_\infty$ | $= \text{ess sup}_{t\in I} \|f(t)\|$ |
| $\mathscr{W}^{k,\infty}(I \to \mathbb{R}^n)$ | the set of $k$-times weakly differentiable functions $f: I \to \mathbb{R}^n$ such that $f, \dots, f^{(k)} \in \mathscr{L}^\infty(I \to \mathbb{R}^n)$ |
| $\mathscr{C}(V \to \mathbb{R}^n)$ | the set of continuous functions $f: V \to \mathbb{R}^n$, $V \subseteq \mathbb{R}^m$ |
| $f\|_W$ | restriction of the function $f: V \to \mathbb{R}^n$ to $W \subseteq V$ |
| $\text{dist}(y,X)$ | $= \inf_{x\in X} \|y-x\|$, the distance of $y \in \mathbb{R}^n$ to a set $X \subseteq \mathbb{R}^n$ |

### B. Framework and system class

In the present paper we consider the framework of one vehicle following another, see Fig. 1.

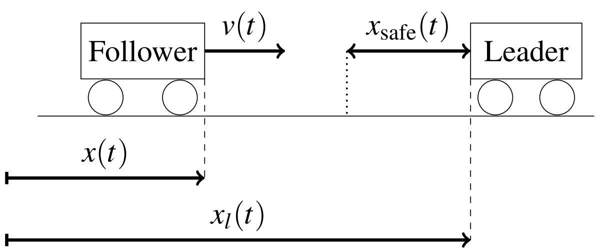


Fig. 1: Vehicle following framework.

By $x_l$ we denote the position of the leader vehicle, while $x$ and $v$ denote the position and velocity of the follower vehicle. The change in momentum of the latter is given by the difference of the force $F$ generated by the contact of the wheels with the road and the forces due to gravity $F_g$ (including the changing slope of the road), the aerodynamic drag $F_a$ and the rolling friction $F_r$. Detailed modelling of these forces can be very complicated since all the individual components of the vehicle have to be taken into account. Therefore, we use the following simple models which are taken from [1, Sec. 3.1]:

$$\begin{aligned}
&F_g: \mathbb{R}_{\geq 0} \to \mathbb{R},\ t \mapsto mg\sin\theta(t),\\
&F_a: \mathbb{R}_{\geq 0} \times \mathbb{R} \to \mathbb{R},\ (t,v) \mapsto \tfrac{1}{2}\rho(t) C_d A v^2,\\
&F_r: \mathbb{R} \to \mathbb{R},\ v \mapsto mgC_r\,\text{sgn}(v),
\end{aligned}$$

where $m$ (in kg) denotes the mass of the (following) vehicle, $g = 9.81\,\text{m/s}^2$ is the acceleration of gravity, $\theta(t) \in [-\frac{\pi}{2}\text{rad}, \frac{\pi}{2}\text{rad}]$ and $\rho(t)$ (in kg/m$^3$) denote the slope of the road and the (bounded) density of air at time $t$, resp., $C_d$ denotes the (dimensionless) shape-dependent aerodynamic drag coefficient and $C_r$ the (dimensionless) coefficient of rolling friction, and $A$ (in m$^2$) is the frontal area of the vehicle.

Since the discontinuous nature of the rolling friction causes some problems in the theoretical treatment we approximate the sgn function by the smooth error function

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2}\,\mathrm{d}t\,, \quad z \in \mathbb{R},$$

using the property that

$$\forall z \in \mathbb{R}: \lim_{\alpha\to\infty} \text{erf}(\alpha z) = \text{sgn}(z).$$

Therefore, we will use the following model for the rolling friction:

$$F_r: \mathbb{R} \to \mathbb{R},\ v \mapsto mgC_r\,\text{erf}(\alpha v) \tag{1.1}$$

for sufficiently large parameter $\alpha > 0$.

The force $F$ which is generated by the engine of the vehicle is usually given as torque curve (depending on the engine speed) times a signal which controls the throttle position, see [1, Sec. 3.1]. Since the latter can be calculated from any given force $F$ and velocity $v$ (taking the current gear into account), here we assume that we can directly control the force $F$, i.e., the control signal is $u(t) = F(t)$. The equations of motion for the vehicle are then given by

$$\begin{aligned}
\dot{x}(t) &= v(t),\\
m\dot{v}(t) &= u(t) - F_g(t) - F_a\big(t, v(t)\big) - F_r\big(v(t)\big),
\end{aligned} \tag{1.2}$$

with the initial conditions

$$x(0) = x^0 \in \mathbb{R}, \quad v(0) = v^0 \in \mathbb{R}. \tag{1.3}$$

### C. Control objective

Roughly speaking, the control objective is to design a control input $u(t)$ such that $v(t)$ is as close to a given favourite speed $v_{\text{ref}}(t)$ as possible, while at the same time a safety distance to the leading vehicle is guaranteed, i.e., $x_l(t) - x(t) \geq x_{\text{safe}}(t)$. The safety distance $x_{\text{safe}}(t)$ should prevent collision with the leading vehicle and is typically a function of the vehicle velocity, but could also be a constant or a function of other variables. In the literature different concepts are used, see e.g. [22], [23] and the references therein. A common model for the safety distance that we use in the present paper is

$$x_{\text{safe}}(t) = \lambda_1 v(t) + \lambda_2 \tag{1.4}$$

with positive constants $\lambda_1$ (in s) and $\lambda_2$ (in m). The parameter $\lambda_1$ models the time gap between the leader and follower vehicle and $\lambda_2$ is the minimal distance when the velocity is zero. If for instance $\lambda_1 = 0.5\,\text{s}$, then it would take the following vehicle $0.5\,\text{s}$ to arrive at the leading vehicle's present position.

We assume that the distance $x_l(t) - x(t)$ to the leader vehicle as well as the velocity $v(t)$ can be measured, i.e., they are available for the controller design. Apart from that, the controller design should be model-free, i.e., knowledge of the parameters $m, \theta(t), \rho(t), C_d, C_r$ and $A$ as well as of the initial values $x^0, v^0$ is not required. This makes the controller robust to modelling errors, uncertainties, noise and disturbances. Summarizing, the objective is to design a (nonlinear and time-varying) control law of the form

$$u(t) = F\big(t, v(t), x_l(t) - x(t)\big) \tag{1.5}$$

such that, when applied to a system (1.2), in the closed-loop system we have that for all $t \geq 0$

(O1) $x_l(t) - x(t) \geq x_{\text{safe}}(t)$,
(O2) $|v(t) - v_{\text{ref}}(t)|$ is as small as possible such that (O1) is not violated.

### D. Funnel control for relative degree one systems

The final control design will consist of two different funnel controllers for appropriate relative degree one systems. The first version of the funnel controller was developed in [12] and this version will be sufficient for our purposes. We consider nonlinear relative degree one systems governed by functional differential equations of the form

$$\begin{aligned} \dot y(t) &= f\big(d(t), (Ty)(t)\big) + \gamma u(t), \\ y|_{[-h,0]} &= y^0 \in \mathscr{C}([-h,0] \to \mathbb{R}), \end{aligned} \tag{1.6}$$

where $h \geq 0$ is the "memory" of the system, $\gamma > 0$ is the high-frequency gain and

- $d \in \mathscr{L}^\infty(\mathbb{R}_{\geq 0} \to \mathbb{R}^p)$, $p \in \mathbb{N}$, is a disturbance;
- $f \in \mathscr{C}(\mathbb{R}^p \times \mathbb{R}^q \to \mathbb{R})$, $q \in \mathbb{N}$;
- $T : \mathscr{C}([-h,\infty) \to \mathbb{R}) \to \mathscr{L}^\infty_{\text{loc}}(\mathbb{R}_{\geq 0} \to \mathbb{R}^q)$ is an operator with the following properties:
  a) $T$ maps bounded trajectories to bounded trajectories, i.e, for all $c_1 > 0$, there exists $c_2 > 0$ such that for all $\zeta \in \mathscr{C}([-h,\infty) \to \mathbb{R})$,
  $$\sup_{t\in[-h,\infty)} \|\zeta(t)\| \leq c_1 \Rightarrow \sup_{t\in[0,\infty)} \|T(\zeta)(t)\| \leq c_2.$$
  b) $T$ is causal, i.e., for all $t \geq 0$ and all $\zeta, \xi \in \mathscr{C}([-h,\infty) \to \mathbb{R})$:
  $$\zeta|_{[-h,t]} = \xi|_{[-h,t]} \implies T(\zeta)|_{[0,t]} \overset{\text{a.e.}}{=} T(\xi)|_{[0,t]},$$
  where "a.e." stands for "almost everywhere".
  c) $T$ is locally Lipschitz continuous in the following sense: for all $t \geq 0$ there exist $\tau, \delta, c > 0$ such that for all $\zeta, \Delta\zeta \in \mathscr{C}([-h,\infty) \to \mathbb{R})$ with $\Delta\zeta|_{[-h,t]} = 0$ and $\| \Delta\zeta|_{[t,t+\tau]} \|_\infty < \delta$ we have
  $$\Big\| \big(T(\zeta + \Delta\zeta) - T(\zeta)\big)\big|_{[t,t+\tau]} \Big\|_\infty \leq c \| \Delta\zeta|_{[t,t+\tau]} \|_\infty.$$

The funnel controller for systems (1.6) is of the form

$$\boxed{\begin{aligned} u(t) &= -k(t)e(t), \qquad e(t) = y(t) - y_{\text{ref}}(t), \\ k(t) &= \frac{1}{1 - \varphi(t)^2 e(t)^2}, \end{aligned}} \tag{1.7}$$

where $y_{\text{ref}} \in \mathscr{W}^{1,\infty}(\mathbb{R}_{\geq 0} \to \mathbb{R})$ is the reference signal, and guarantees that the tracking error $e(t)$ evolves within a prescribed performance funnel

$$\mathscr{F}_\varphi := \{\ (t,e) \in \mathbb{R}_{\geq 0} \times \mathbb{R} \ \mid\ \varphi(t)|e| < 1\ \}, \tag{1.8}$$

which is determined by a function $\varphi$ belonging to

$$\Phi := \left\{ \varphi \in \mathscr{W}^{1,\infty}(\mathbb{R}_{\geq 0} \to \mathbb{R}) \,\middle|\, \begin{array}{l} \varphi(s) > 0 \text{ for all } s > 0 \text{ and} \\ \text{for all } \varepsilon > 0\text{:} \\ (1/\varphi)|_{[\varepsilon,\infty)} \in \mathscr{W}^{1,\infty}([\varepsilon,\infty) \to \mathbb{R}) \end{array} \right\}.$$

The funnel boundary is given by the reciprocal of $\varphi$, see Fig. 2. The case $\varphi(0) = 0$ is explicitly allowed, meaning that no restriction is put on the initial value since $\varphi(0)|e(0)| < 1$; the funnel boundary $1/\varphi$ has a pole at $t = 0$ in this case.

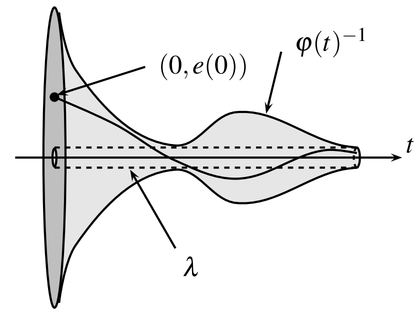


Fig. 2: Error evolution in a funnel $\mathscr{F}_\varphi$ with boundary $\varphi(t)^{-1}$.

An important property is that each performance funnel $\mathscr{F}_\varphi$ is bounded away from zero, i.e., because of boundedness of $\varphi$ there exists $\lambda > 0$ such that $1/\varphi(t) \geq \lambda$ for all $t > 0$. We stress that the funnel boundary is not necessarily monotonically decreasing. Widening the funnel over some later time interval might be beneficial, e.g., when periodic disturbances are present or the reference signal changes strongly. For typical choices of funnel boundaries see e.g. [24, Sec. 3.2].

In [12], the existence of global solutions of the closed-loop system (1.6), (1.7) is investigated. To this end, $y : [-h,\omega) \to \mathbb{R}$ is called a *solution* of (1.6), (1.7) on $[-h,\omega)$, $\omega \in (0,\infty]$, if $y|_{[-h,0]} = y^0$ and $y|_{[0,\omega)}$ is weakly differentiable and satisfies (1.6), (1.7) for almost all $t \in [0,\omega)$; $y$ is called *maximal*, if it has no right extension that is also a solution. Note that uniqueness of solutions of (1.6), (1.7) is not guaranteed in general.

The following result is proved in [12].

*Theorem 1.1:* Consider a system (1.6) with initial trajectory $y^0 \in \mathscr{C}([-h,0] \to \mathbb{R})$, a reference signal $y_{\text{ref}} \in \mathscr{W}^{1,\infty}(\mathbb{R}_{\geq 0} \to \mathbb{R})$ and a funnel function $\varphi \in \Phi$ such that

$$\varphi(0)|y^0(0) - y_{\text{ref}}(0)| < 1.$$

Then the controller (1.7) applied to (1.6) yields a closed-loop system which has a solution, and every maximal solution $y : [0,\omega) \to \mathbb{R}$ has the properties:

(i) $\omega = \infty$;
(ii) all involved signals $y(\cdot), k(\cdot)$ and $u(\cdot)$ are bounded;
(iii) the tracking error evolves uniformly within the performance funnel in the sense
$$\exists\, \varepsilon > 0\ \forall\, t > 0 :\ |e(t)| \leq \varphi(t)^{-1} - \varepsilon.$$

### E. Organization of the present paper

In Section II we present a novel funnel cruise controller which satisfies the control objectives as stated in Section I-C. The controller is basically the conjunction of a velocity funnel controller and a distance funnel controller, both formulated for appropriate relative degree one systems. Those controllers are presented separately before the final controller design is stated and feasibility is proved. In Section III the performance of the controller is illustrated for some typical model parameters and scenarios from daily traffic. Some conclusions are given in Section IV.

## II. Funnel cruise control

In this section we present our novel funnel cruise control design to achieve (O1) and (O2), which consists of a velocity funnel controller and a distance funnel controller. We first present those controllers separately before we state the unified controller design.

### A. Velocity funnel controller

When the leader vehicle is far away we do not need to care about the control objective (O1) and simply need to regulate the velocity $v$ to the favourite velocity $v_{\mathrm{ref}} \in \mathscr{W}^{1,\infty}(\mathbb{R}_{\geq 0} \to \mathbb{R})$. For this purpose we can treat the velocity $v$ as the output of system (1.2) and hence the velocity tracking error is given by $e_v(t) = v(t) - v_{\mathrm{ref}}(t)$. Since the first equation in (1.2) can be ignored in this case (it does not play a role for the input-output behavior), we may define $d(t) := \big(F_g(t), \rho(t)\big)$ for $t \geq 0$ and

$$f_v : \mathbb{R}^3 \to \mathbb{R},\ (d_1, d_2, v) \mapsto \tfrac{1}{m}\left(d_1 + \tfrac{1}{2} d_2 C_d A v^2 + F_r(v)\right).$$

Since $\rho$ is assumed to be bounded we obtain that $d$ is bounded and the second equation in (1.2) can be written as

$$\dot v(t) = \tfrac{1}{m} u(t) - f_v\big(d(t), v(t)\big), \tag{2.1}$$

and hence belongs to the class of systems (1.6) with the identity operator $Tv = v$. Then Theorem 1.1 yields feasibility of the velocity funnel controller

$$\begin{aligned} u_v(t) &= -k_v(t) e_v(t), \qquad e_v(t) = v(t) - v_{\mathrm{ref}}(t),\\ k_v(t) &= \frac{1}{1 - \varphi_v(t)^2 e_v(t)^2}, \end{aligned} \tag{2.2}$$

where $\varphi_v \in \Phi$, when applied to system (1.2) with initial conditions (1.3) such that $\varphi_v(0)|v^0 - v_{\mathrm{ref}}(0)| < 1$.

We stress that since $x$ has been ignored for the controller design above, the first equation in (1.2) may cause it to grow unboundedly. However, since $x_l$ is assumed to be bounded, $x_l - x$ will eventually get small enough so that the distance funnel controller discussed in the following section becomes active. In the end, this will guarantee boundedness of $x$.

### B. Distance funnel controller

If the leader vehicle is close, then the main objective of the controller is to ensure that (O1) is guaranteed, so that $v(t)$ may be much smaller than $v_{\mathrm{ref}}(t)$ if necessary. To this end, we introduce a performance funnel, defined by $\varphi_d \in \Phi$ with $\varphi_d(0) \neq 0$, which lies directly in front of the safety distance to the leader vehicle, see Fig. 3.

The aim is then to regulate the position $x(t)$ to the middle of this performance funnel given by $x_l(t) - x_{\mathrm{safe}}(t) - \varphi_d(t)^{-1}$,

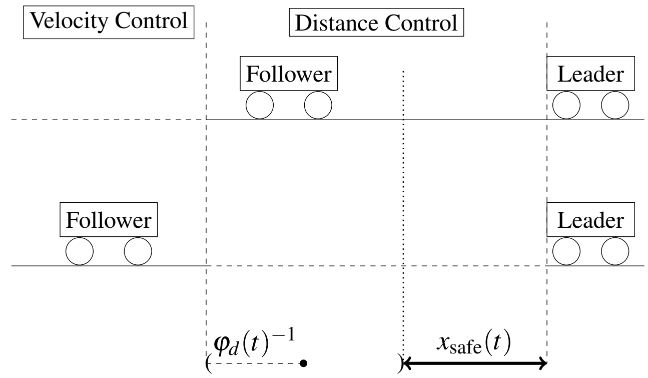


Fig. 3: Illustration of the distance funnel controller.

where $x_l \in \mathscr{W}^{1,\infty}(\mathbb{R}_{\geq 0} \to \mathbb{R})$. The corresponding distance tracking error is hence given by

$$e_d(t) = x(t) - x_l(t) + x_{\mathrm{safe}}(t) + \varphi_d(t)^{-1}.$$

In order to reformulate system (1.2) in the form (1.6) with appropriate output $y$ and reference signal $y_{\mathrm{ref}}$ we recall that $x_{\mathrm{safe}}(t) = \lambda_1 v(t) + \lambda_2$ and define

$$\begin{aligned} y(t) &:= \lambda_1 v(t) + x(t),\\ y_{\mathrm{ref}}(t) &:= x_l(t) - \lambda_2 - \varphi_d(t)^{-1}. \end{aligned}$$

Then $e_d(t) = y(t) - y_{\mathrm{ref}}(t)$ and we further find that, invoking the first equation in (1.2),

$$\dot x(t) = -\tfrac{1}{\lambda_1} x(t) + \tfrac{1}{\lambda_1} y(t), \quad x(0) = x^0,$$

hence

$$x(t) = e^{-\frac{1}{\lambda_1} t} x^0 + \int_0^t \tfrac{1}{\lambda_1} e^{-\frac{1}{\lambda_1}(t-s)} y(s)\,\mathrm{d}s =: (T_1 y)(t), \quad t \geq 0.$$

Now define

$$Ty := \big(T_1 y, y\big)^\top$$

for all $y \in \mathscr{C}(\mathbb{R}_{\geq 0} \to \mathbb{R})$. It is straightforward to check that the operator $T : \mathscr{C}([0,\infty) \to \mathbb{R}) \to \mathscr{L}^\infty_{\mathrm{loc}}(\mathbb{R}_{\geq 0} \to \mathbb{R}^2)$, which is parameterized by $x^0 \in \mathbb{R}$, has the properties a)–c) as stated in Section I-D. Using equation (2.1) as well as $d$ and $f_v$ defined in Section II-A we obtain

$$\begin{aligned} \dot y(t) &= \lambda_1 \dot v(t) + \dot x(t)\\ &= \tfrac{\lambda_1}{m} u(t) - \lambda_1 f_v\big(d(t), v(t)\big) - \tfrac{1}{\lambda_1} x(t) + \tfrac{1}{\lambda_1} y(t),\\ &= \tfrac{\lambda_1}{m} u(t) - \lambda_1 f_v\left(d(t), -\tfrac{1}{\lambda_1}(T_1 y)(t) + \tfrac{1}{\lambda_1} y(t)\right)\\ &\quad - \tfrac{1}{\lambda_1}(T_1 y)(t) + \tfrac{1}{\lambda_1} y(t),\\ &= \tfrac{\lambda_1}{m} u(t) - f_d\big(d(t), (Ty)(t)\big), \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} &f_d : \mathbb{R}^4 \to \mathbb{R},\\ &(d_1, d_2, \zeta_1, \zeta_2) \mapsto \lambda_1 f_v\left(d_1, d_2, -\tfrac{1}{\lambda_1}\zeta_1 + \tfrac{1}{\lambda_1}\zeta_2\right) + \tfrac{1}{\lambda_1}\zeta_1 - \tfrac{1}{\lambda_1}\zeta_2. \end{aligned}$$

Clearly, (2.3) belongs to the class of systems (1.6). Then Theorem 1.1 yields feasibility of the distance funnel controller

$$u_d(t) = -k_d(t)e_d(t), \qquad e_d(t) = x(t) - x_l(t)$$
$$k_d(t) = \frac{1}{1-\varphi_d(t)^2 e_d(t)^2}, \qquad + x_{\text{safe}}(t) + \varphi_d(t)^{-1}, \tag{2.4}$$

when applied to system (1.2) with initial conditions (1.3) such that $\varphi_d(0)|\lambda_1 v^0 + x^0 - x_l(0) + \lambda_2 + \varphi_d(0)^{-1}| < 1$.

### C. Final control design and its feasibility

In Sections II-A and II-B we have seen that the separate velocity and distance funnel controllers are feasible when the initial conditions lie within the funnel boundaries at $t = 0$; the control objective (O1) is ignored in the case of velocity control and the control objective (O2) is ignored in the case of distance control. However, it is our aim to simultaneously satisfy (O1) and (O2), i.e., always guarantee the safety distance and regulate the velocity to the favourite velocity as far as possible. This means that two additional scenarios must be possible:

- if the follower vehicle, while using the velocity funnel controller (2.2), enters the performance funnel in front of the safety distance, i.e., $x(t) = x_l(t) - x_{\text{safe}}(t) - \varphi_d(t)^{-1}$, then the controller should switch to the distance funnel controller (2.4);
- when the distance funnel controller is active it should still be guaranteed that $v(t) < v_{\text{ref}}(t) + \varphi_v(t)^{-1}$, but it is possible that $v(t) \le v_{\text{ref}}(t) - \varphi_v(t)^{-1}$ when the leader decelerates; in the latter case it is not possible to switch back to (2.2).

A controller which combines (2.2) and (2.4) and takes the above conditions into account faces an immediate problem: The controller (2.4) has a singularity when $x(t) = x_l(t) - x_{\text{safe}}(t) - \varphi_d(t)^{-1}$ since $k_d(t) \nearrow \infty$ at such points. Likewise, $k_v(t) \nearrow \infty$ for points where $v(t) = v_{\text{ref}}(t) - \varphi_v(t)^{-1}$, i.e., when a strong deceleration is necessary. To resolve these problems, the minimum of the control signals $u_v(t)$ and $u_d(t)$ is chosen in the region where the velocity performance funnel and the distance performance funnel intersect, i.e., when $(t, e_v(t)) \in \mathscr{F}_{\varphi_v}$ and $(t, e_d(t)) \in \mathscr{F}_{\varphi_d}$; see also Fig. 4 for an illustration.

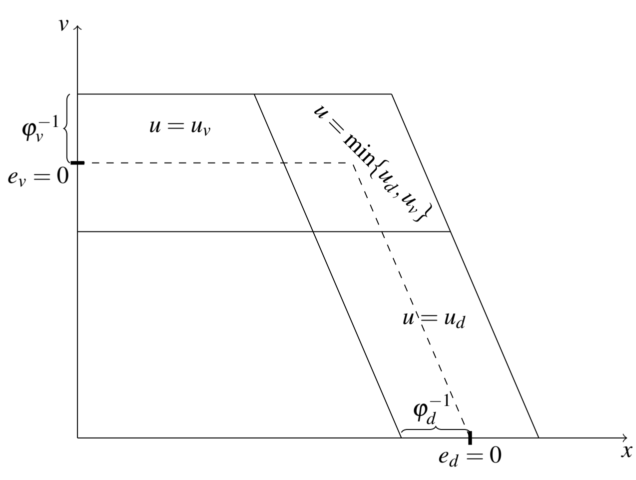


Fig. 4: Illustration of the final control design.

The overall funnel cruise controller is of the form (1.5) and given in (2.5). For later purposes we also define the relatively open set $\mathscr{D}$ given in (2.6).

In the remainder of this section we give the feasibility result for the controller (2.5) and sketch its proof.

*Theorem 2.1:* Consider a system (1.2) with initial conditions (1.3), a favourite velocity $v_{\text{ref}} \in \mathscr{W}^{1,\infty}(\mathbb{R}_{\ge 0} \to \mathbb{R})$, position of the leader vehicle $x_l \in \mathscr{W}^{1,\infty}(\mathbb{R}_{\ge 0} \to \mathbb{R})$, safety distance $x_{\text{safe}}$ as in (1.4) with $\lambda_1, \lambda_2 > 0$ and funnel functions $\varphi_v, \varphi_d \in \Phi$ such that $\varphi_d(0) \neq 0$ and

$$\big(0, x^0, v^0\big) \in \mathscr{D}.$$

Then the funnel cruise controller (2.5) applied to (1.2) yields a closed-loop system which has a solution, and every maximal solution $(x, v) : [0, \omega) \to \mathbb{R}^2$ has the properties:

(i) $\omega = \infty$;
(ii) all involved signals $x(\cdot), v(\cdot)$ and $u(\cdot)$ are bounded;
(iii) the solution evolves uniformly within the set $\mathscr{D}$, i.e.,

$$\exists\, \varepsilon > 0\ \forall\, t \ge 0 :\ \operatorname{dist}\big((t, x(t), v(t)), \partial\mathscr{D}\big) > \varepsilon,$$

where $\partial\mathscr{D}$ denotes the boundary of $\mathscr{D}$ in $\mathbb{R}_{\ge 0} \times \mathbb{R}^2$.

*Sketch of the Proof:* Since the input $u$ as in (2.5) is continuous in $(x, v)$ on the set $\mathscr{D}$ from (2.6), the existence of a maximal solution of the closed-loop system is a consequence of Carathéodory's existence theorem, see e.g. [25, § 10, Thm. XX]. Let $(x, v) : [0, \omega) \to \mathbb{R}^2$, $\omega \in (0, \infty]$, be a maximal solution, then we may also infer that the closure of the graph of $(x, v)$ is not a compact subset of $\mathscr{D}$. Clearly, $x$ and $v$ are bounded since $\varphi_d^{-1}$, $x_l$ and $v_{\text{ref}}$ are bounded and $(t, x(t), v(t)) \in \mathscr{D}$ for all $t \in [0, \omega)$. Therefore, if (iii) holds on $[0, \omega)$, then it implies (i) and (ii).
In order to show (iii) we consider three different cases according to the three parts in the definition of $\mathscr{D}$ in (2.6), see also Fig. 4. By standard arguments as used in the proof of Theorem 1.1 (see [12]) we may exclude that the graph of $(x, v)$ can reach the boundary of $\mathscr{D}$ in the first two cases; in other words, $k_v$ is bounded on the first part where $u = u_v$ and $k_d$ is bounded on the second part where $u = u_d$. At the boundary between the first part and the third part $u_d$ has a pole, and at the boundary between the second part and the third part $u_v$ has a pole; but $u$ is continuous since $u = \min\{u_v, u_d\}$. To show that in this last part the graph of $(x, v)$ cannot reach the boundary of $\mathscr{D}$ we divide it again into four distinct parts and consider the minimum separately on each of them. Using similar arguments as before, the result can then be obtained. ■

## III. Simulations

We illustrate the funnel cruise controller (2.5) for three different scenarios which may occur in daily traffic. The first standard scenario is that the follower vehicle, with a constant favourite velocity $v_{\text{ref}}$, is far away from the leader, catches up and follows it for some time until the leader accelerates past $v_{\text{ref}}$. The second scenario illustrates that safety is guaranteed even in the case of a full brake of the leader vehicle. In the last scenario the leader vehicle has a strongly varying acceleration.

$$
\boxed{\begin{aligned}
u(t) &= \begin{cases} -k_v(t)e_v(t), & e_d(t) \le -\varphi_d(t)^{-1} \ \wedge \ (t, e_v(t)) \in \mathscr{F}_{\varphi_v}, \\ -k_d(t)e_d(t), & e_v(t) \le -\varphi_v(t)^{-1} \ \wedge \ (t, e_d(t)) \in \mathscr{F}_{\varphi_d}, \\ \min\{-k_v(t)e_v(t), -k_d(t)e_d(t)\}, & (t, e_v(t)) \in \mathscr{F}_{\varphi_v} \ \wedge \ (t, e_d(t)) \in \mathscr{F}_{\varphi_d}, \end{cases} \\
k_v(t) &= \frac{1}{1-\varphi_v(t)^2 e_v(t)^2}, \quad e_v(t) = v(t) - v_{\mathrm{ref}}(t), \\
k_d(t) &= \frac{1}{1-\varphi_d(t)^2 e_d(t)^2}, \quad e_d(t) = x(t) - x_l(t) + x_{\mathrm{safe}}(t) + \varphi_d(t)^{-1}.
\end{aligned}} \tag{2.5}
$$

$$
\mathscr{D} := \left\{ (t,x,v) \in \mathbb{R}_{\ge 0} \times \mathbb{R}^2 \ \middle| \ \begin{aligned} & \big(\lambda_1 v + x \le x_l(t) - \lambda_2 - 2\varphi_d(t)^{-1} \ \wedge \ \big(t, v - v_{\mathrm{ref}}(t)\big) \in \mathscr{F}_{\varphi_v}\big) \\ \vee \ & \big(v \le v_{\mathrm{ref}}(t) - \varphi_v(t)^{-1} \ \wedge \ \big(t, \lambda_1 v + x - x_l(t) + \lambda_2 + \varphi_d(t)^{-1}\big) \in \mathscr{F}_{\varphi_d}\big) \\ \vee \ & \big(\big(t, v - v_{\mathrm{ref}}(t)\big) \in \mathscr{F}_{\varphi_v} \ \wedge \ \big(t, \lambda_1 v + x - x_l(t) + \lambda_2 + \varphi_d(t)^{-1}\big) \in \mathscr{F}_{\varphi_d}\big) \end{aligned} \right\}. \tag{2.6}
$$

| $m$ | $\theta(t)$ | $\rho(t)$ | $C_d$ | $C_r$ | $A$ |
|---|---|---|---|---|---|
| $1300\,\mathrm{kg}$ | $0\,\mathrm{rad}$ | $1.3\,\mathrm{kg/m^3}$ | $0.32$ | $0.01$ | $2.4\,\mathrm{m}^2$ |

TABLE I: Parameter values for the vehicle model.

For the simulations we will use some typical parameter values for the model (1.2) which are taken from [1] and summarized in Table I. For the approximated friction model (1.1) we choose the parameter $\alpha = 100$. The initial conditions (1.3) are chosen as $x^0 = 0\,\mathrm{m}$ and $v^0 = 15\,\mathrm{m\,s^{-1}}$ and the constants in (1.4) as $\lambda_1 = 0.5\,\mathrm{s}$ and $\lambda_2 = 2\,\mathrm{m}$. For all three scenarios we choose the favourite velocity $v_{\mathrm{ref}}(t) = 36\,\mathrm{m\,s^{-1}}$ and the funnel functions

$$\varphi_v(t) = \big(22.5e^{-0.2t} + 0.2\big)^{-1}, \quad \varphi_d(t) = 0.25.$$

**Scenario 1**: We have chosen $x_l$ and $v_l = \dot{x}_l$ so that initially the leader vehicle has a larger velocity than the follower, which is hence free to accelerate and catch up using the velocity funnel controller. When the distance is between $x_{\mathrm{safe}}(t) + 2\varphi_d(t)^{-1}$ and $x_{\mathrm{safe}}(t)$, the distance funnel controller will ensure that the safety distance is not violated. After a period of safe following, where $v(t) < v_{\mathrm{ref}}(t) - \varphi_v(t)^{-1}$, the leader accelerates to a velocity larger than $v_{\mathrm{ref}}(t)$ and the velocity funnel controller will again take over.

The simulation of the controller (2.5) for the system (1.2) with parameters as in Table I and the above described scenario over the time interval $0-50$s has been performed in MATLAB (solver: `ode15s`, rel. tol.: $10^{-10}$, abs. tol.: $10^{-10}$) and is depicted in Fig. 5. Fig. 5a shows the distance $x_l - x$ to the leader and the distance funnel in front of the safety distance. The velocities $v$ and $v_l$ are depicted in Fig. 5b together with the velocity funnel. Fig. 5c shows the input signal $u$ generated by the controller and the engine force $u_l$ of the leader vehicle. We stress that, due to the mass of the vehicles of $1300\,\mathrm{kg}$, the forces $u$ and $u_l$ which are between $\pm 10^4$N correspond to an acceleration between $\pm 8\,\mathrm{m/s^2}$. It can be seen that the controller achieves the favourite velocity as far as possible while guaranteeing safety at all times, thus the control objectives (O1) and (O2) are satisfied.

**Scenario 2**: We have chosen $x_l$ and $v_l$ so that after a period of safe following the leader vehicle suddenly fully brakes. This illustrates that even in such extreme cases the funnel cruise controller is able to guarantee that the safety distance is not violated.

The simulation of the controller (2.5) for the system (1.2) with parameters as in Table I and the above described scenario over the time interval $0-50$s has been performed in MATLAB (solver: `ode15s`, rel. tol.: $10^{-10}$, abs. tol.: $10^{-10}$) and is depicted in Fig. 6. It can be seen in Fig. 6a that the safety distance is always guaranteed. From Fig. 6b we can observe that the velocity $v$ of the follower does not drop as sharp as the velocity $v_l$ of the leader. Fig. 6c shows that the engine forces $u$ and $u_l$ are quite comparable.

**Scenario 3**: We have chosen $x_l$ and $v_l$ so that the leader vehicle has a strongly varying acceleration. After a period of velocity control where the follower vehicle is free to accelerate close to the favourite velocity $v_{\mathrm{ref}}$, it will catch up with the leader vehicle and a period of safe following using distance funnel control follows. During this period several (sharp) acceleration and deceleration maneuvers are necessary to guarantee safety in the face of the mercurial behavior of the leader vehicle.

The simulation of the controller (2.5) for the system (1.2) with parameters as in Table I and the above described scenario over the time interval $0-50$s has been performed in MATLAB (solver: `ode15s`, rel. tol.: $10^{-10}$, abs. tol.: $10^{-10}$) and is depicted in Fig. 7. It can be seen in Fig. 7a that safety is guaranteed even in the case of the strongly varying behavior of the leader. The velocity $v$ of the follower, as shown in Fig. 7b, does not vary as strongly as $v_l$, but shows a much smoother behavior. The engine force $u$ depicted in Fig. 7c shows sharp drops and rises, but this cannot be avoided since the funnel cruise controller (2.5) is causal and hence cannot look into the future. This is different from other approaches such as MPC (see e.g. [6], [7], [8]) which is able to incorporate future information since an optimal control problem is solved over some future time interval. However, the drawback of this is that the model (1.2) must be known

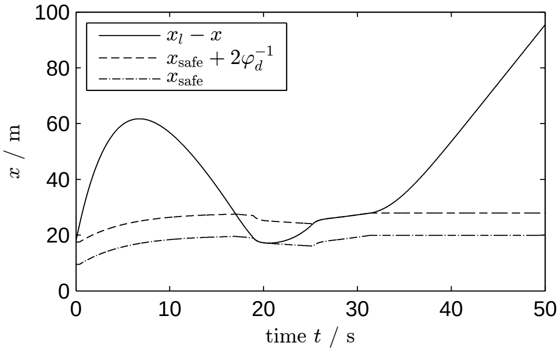


Fig. 5a: Distance to leader and distance funnel

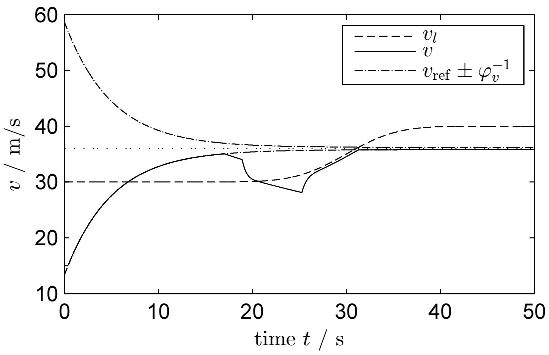


Fig. 5b: Velocities and velocity funnel

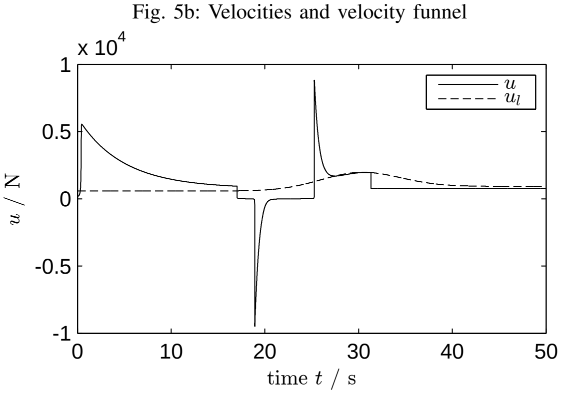


Fig. 5c: Input and leader engine force

Fig. 5: Simulation of the funnel cruise controller (2.5) for the system (1.2) with parameters as in Table I in Scenario 1.

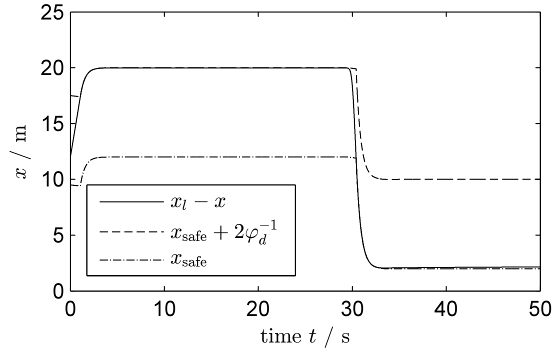


Fig. 6a: Distance to leader and distance funnel

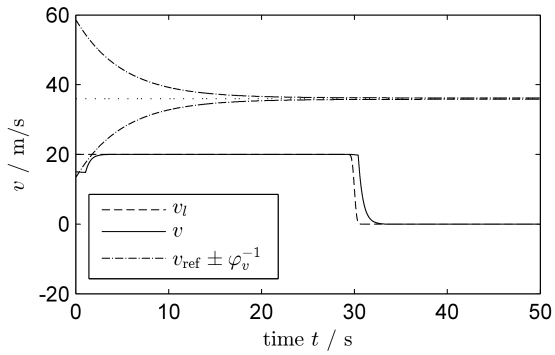


Fig. 6b: Velocities and velocity funnel

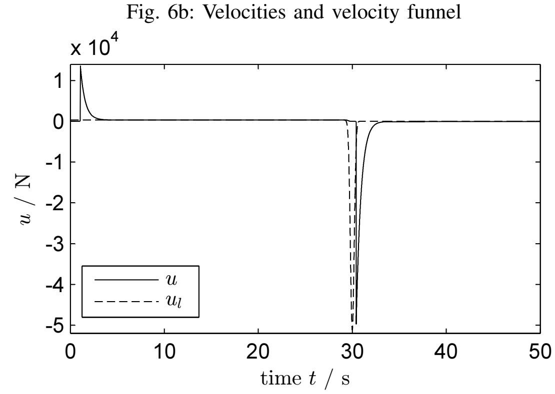


Fig. 6c: Input and leader engine force

Fig. 6: Simulation of the funnel cruise controller (2.5) for the system (1.2) with parameters as in Table I in Scenario 2.

as good as possible.

## IV. Conclusions

In the present paper we proposed a novel and universal adaptive cruise control mechanism which is model-free and guarantees safety at all times. The funnel cruise controller consists of a velocity funnel controller, which is active when the leader vehicle is far away, and a distance funnel controller, which ensures that the safety distance is not violated when the leader vehicle is close. We have sketched the proof of feasibility of this controller; a comprehensive proof will be given in future works. Three simulation scenarios illustrate the application of the funnel cruise controller.

The simulations show that, although the funnel cruise controller satisfies the control objectives, the generated control input (engine force of the follower vehicle) usually contains sharp peaks which are not desired in terms of driver comfort. This issue should be resolved by smoothing the peaks e.g. by combining the funnel cruise controller with the PI-funnel controller with anti-windup as discussed in [26]. Furthermore, constraints on the acceleration should be incorporated in future research to avoid unrealistic high peaks in the control input generated by the controller.

Another topic of future research is the investigation of platoons of several vehicles, each equipped with a funnel cruise controller. An important question is under which

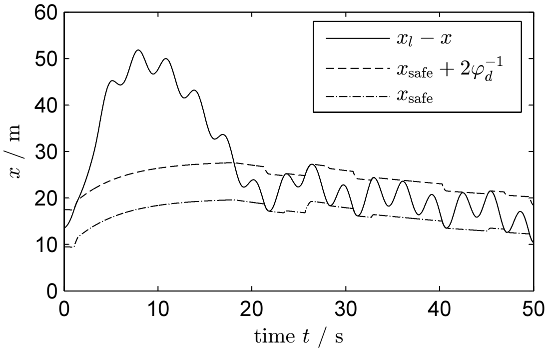


Fig. 7a: Distance to leader and distance funnel

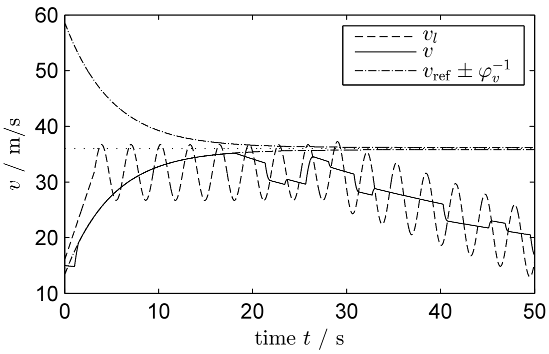


Fig. 7b: Velocities and velocity funnel

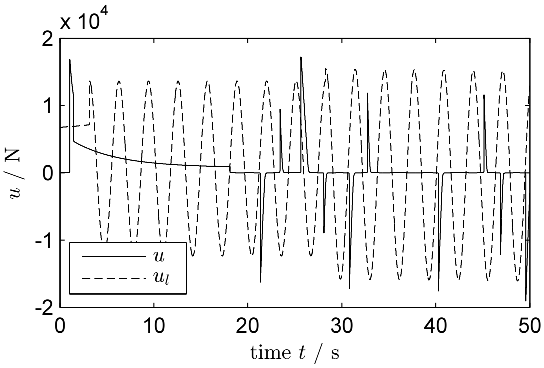


Fig. 7c: Input and leader engine force

Fig. 7: Simulation of the funnel cruise controller (2.5) for the system (1.2) with parameters as in Table I in Scenario 3.

conditions string stability of the platoon is achieved and whether some communication between the vehicles must be allowed for this.

## REFERENCES


[1] K. J. Åström and R. M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton, NJ: Princeton University Press, 2008.

[2] L. Xiao and F. Gao, "A comprehensive review of the development of adaptive cruise control systems," *Vehicle System Dynamics*, vol. 48, no. 10, pp. 1167–1192, 2010.

[3] P. A. Ioannou and C. C. Chien, "Autonomous intelligent cruise control," *IEEE Trans. Vehicular Technology*, vol. 42, no. 4, pp. 657–672, 1993.

[4] P. Ioannou, Z. Xu, S. Eckert, D. Clemons, and T. Sieja, "Intelligent cruise control: Theory and experiment," in *Proc. 32nd IEEE Conf. Decis. Control*, 1993, pp. 1885–1890.

[5] D. Yanakiev and I. Kanellakopoulos, "Nonlinear spacing policies for automated heavy-duty vehicles," *IEEE Trans. Vehicular Technology*, vol. 47, no. 4, pp. 1365–1377, 1998.

[6] V. L. Bageshwar, W. L. Garrard, and R. Rajamani, "Model predictive control of transitional maneuvers for adaptive cruise control vehicles," *IEEE Trans. Vehicular Technology*, vol. 53, no. 5, pp. 1573–1885, 2004.

[7] S. Li, K. Li, R. Rajamani, and J. Wang, "Model predictive multi-objective vehicular adaptive cruise control," *IEEE Trans. Control Systems Technology*, vol. 19, no. 3, pp. 556–566, 2011.

[8] S. Magdici and M. Althoff, "Adaptive cruise control with safety guarantees for autonomous vehicles," *IFAC PapersOnLine*, vol. 50, no. 1, pp. 5774–5781, 2017.

[9] A. Ames, J. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs with application to adaptive cruise control," in *Proc. 53rd IEEE Conf. Decis. Control, Los Angeles, USA*, 2014, pp. 6271–6278.

[10] A. Mehra, W. L. Ma, F. Berg, P. Tabuada, J. Grizzle, and A. Ames, "Adaptive cruise control: Experimental validation of advanced controllers on scale-model cars," in *Proc. American Control Conf. 2015*, 2015, pp. 1411–1418.

[11] P. Nilsson, O. Hussien, A. Balkan, Y. Chen, A. Ames, J. Grizzle, N. Ozay, H. Peng, and P. Tabuada, "Correct-by-construction adaptive cruise control: Two approaches," *IEEE Trans. Control Systems Technology*, vol. 24, no. 4, pp. 1294–1307, 2016.

[12] A. Ilchmann, E. P. Ryan, and C. J. Sangwin, "Tracking with prescribed transient behaviour," *ESAIM: Control, Optimisation and Calculus of Variations*, vol. 7, pp. 471–493, 2002.

[13] A. Ilchmann and E. P. Ryan, "High-gain control without identification: a survey," *GAMM Mitt.*, vol. 31, no. 1, pp. 115–125, 2008.

[14] T. Berger, L. H. Hoàng, and T. Reis, "Funnel control for nonlinear systems with known strict relative degree," *Automatica*, vol. 87, pp. 345–357, 2018.

[15] A. Ilchmann and S. Trenn, "Input constrained funnel control with applications to chemical reactor models," *Syst. Control Lett.*, vol. 53, no. 5, pp. 361–375, 2004.

[16] C. M. Hackl, *Non-identifier Based Adaptive Control in Mechatronics–Theory and Application*, ser. Lecture Notes in Control and Information Sciences. Cham, Switzerland: Springer-Verlag, 2017, vol. 466.

[17] C. M. Hackl, N. Hopfe, A. Ilchmann, M. Mueller, and S. Trenn, "Funnel control for systems with relative degree two," *SIAM J. Control Optim.*, vol. 51, no. 2, pp. 965–995, 2013.

[18] A. Ilchmann and H. Schuster, "PI-funnel control for two mass systems," *IEEE Trans. Autom. Control*, vol. 54, no. 4, pp. 918–923, 2009.

[19] A. Senfelds and A. Paugurs, "Electrical drive DC link power flow control with adaptive approach," in *Proc. 55th Int. Sci. Conf. Power Electr. Engg. Riga Techn. Univ., Riga, Latvia*, 2014, pp. 30–33.

[20] T. Berger and T. Reis, "Zero dynamics and funnel control for linear electrical circuits," *J. Franklin Inst.*, vol. 351, no. 11, pp. 5099–5132, 2014.

[21] A. Pomprapa, S. Weyer, S. Leonhardt, M. Walter, and B. Misgeld, "Periodic funnel-based control for peak inspiratory pressure," in *Proc. 54th IEEE Conf. Decis. Control, Osaka, Japan*, 2015, pp. 5617–5622.

[22] D. Hong, C. Park, Y. Yoo, and S. Hwang, "Advanced smart cruise control with safety distance considered road friction coefficient," *Int. J. Comp. Theory Engin.*, vol. 8, no. 3, pp. 198–202, 2016.

[23] K. Santhanakrishnan and R. Rajamani, "On spacing policies for highway vehicle automation," *IEEE Trans. Intelligent Transp. Systems*, vol. 4, no. 4, pp. 198–204, 2003.

[24] A. Ilchmann, "Decentralized tracking of interconnected systems," in *Mathematical System Theory - Festschrift in Honor of Uwe Helmke on the Occasion of his Sixtieth Birthday*, K. Hüper and J. Trumpf, Eds. CreateSpace, 2013, pp. 229–245.

[25] W. Walter, *Ordinary Differential Equations*. New York: Springer-Verlag, 1998.

[26] C. M. Hackl, "PI-funnel control with anti-windup and its application to speed control of electrical drives," in *Proc. 52nd IEEE Conf. Decis. Control, Florence, Italy*, 2013, pp. 6250–6255.